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METHOD OF ITERATIVE SINGLE-CHANNEL BLIND SEPARATION FOR QPSK SIGNALS

A method for single-channel blind separation of two QPSK (quadrature phase shift keying) signals is proposed. The method is based on the iterative maximization of a posteriori probability for mixture's components. The relations for a posteriori probabilities are derived and on its basis the iterative algorithm for the estimation of mixture's components is developed. The algorithm for the estimation of channel parameters (amplitudes, phases, time delays) is also developed. The effectiveness of method is demonstrated for various noise levels and time diversities between channels. The proposed parameters' estimation procedure provides significant reduction of bit error rate (BER) over the case of unknown parameters.

Key words: *Blind Source Separation, BPSK (Binary Phase Shift Keying), QPSK (Quadrature Phase Shift Keying).*

Introduction. Blind Source Separation is rapidly evolving since 1990s and comprises wide field of problems in telecommunications. There are a lot of existing approaches for the solution of blind source separation problem (see, e.g. [1–6]).

A general statement of blind separation problem is shown at Fig. 1. There are p sources which are mixed by some vector-function at additive noise background. Having m sensors, separation algorithm has to estimate the source signals.

Most of methods imply that the number of sensors is not less than the number of sources. However, the more frequently found case is of one sensor and several sources (underdetermined blind separation problem). When there are less sensors than sources, the problem is known to be underdetermined and turns out to be quite challenging. To remove the indeterminacy, we need to exploit any a priori knowledge induced by the system.

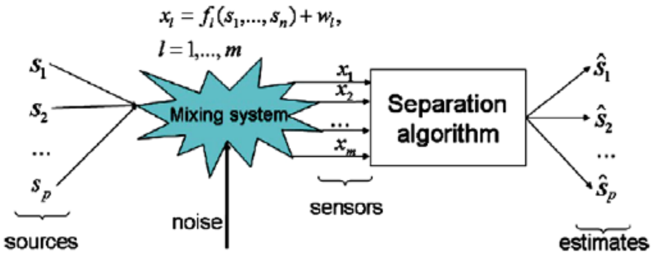

Fig. 1. *General statement of blind source separation problem*

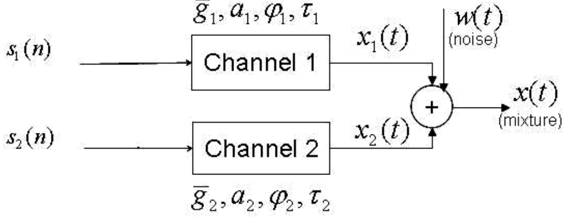

***Fig. 2.** Statement of considered underdetermined BSS problem*

So, consider the problem from radio communications presented at Fig. 2. We have two discrete sequences $s_1(n)$, $s_2(n)$ which are both QPSK, i.e. possess values $\pm 1 \pm j$. They are passed through two independent communication channels. Their mixture $x(t)$ is the observation signal. The goal is to restore original sequences $s_1(n)$, $s_2(n)$.

In this paper the Bayesian approach proposed in [2] for the case of BPSK signals is further developed. We expand this approach for the case of QPSK signals and, besides, add channel parameters estimation procedure, while in work [2] the channel parameters were assumed to be known. Higher order modulations can be handled as well, though computational expenses grow exponentially with the modulation order.

The organization of the paper is as follows. First, the structure of the proposed receiver is described. Then the idea of Bayesian approach to the estimation of original QPSK sequences as well as iterative separation algorithm is explained. The following sections include estimation of channel parameters and the experimental results.

Preliminaries. As is known, in the data communication system, the transmitted QPSK sequences of symbols must be bandlimited using a pulse shaping filter $g(t)$ before transmitting. The received mixture of two digitally modulated signals received by one antenna in single channel can be expressed as:

$$x(t) = x_1(t) + x_2(t) + w(t),$$

where $x_u(t), u = 1,2$ are the signals from two sources:

$$x_u(t) = a_u e^{j\varphi_u} \sum_{n=-\infty}^{\infty} s_u(n) g(t - nT_s - \tau_u), u = 1,2$$

and $s_u(n); u = 1,2$ are original QPSK sequences to be estimated; T_s is a symbol period; a_u are the amplitudes; φ_u are the phases; τ_u are the time shifts. $g(t)$ is a total channel response (assumed to be raised square-root cosine with known roll-off), $w(t)$ is background noise with variance σ^2.

The structure of proposed receiver. In this section we derive the separation algorithm, described in [2], but we do not limit ourselves to BPSK modulation and show that this approach can be applied to any kind of modulation. Structure of proposed receiver is presented at Fig. 3. The idea is to produce two discrete sequences: $y_1(n)$ synchronous with the first source and $y_2(n)$ synchronous with second source.

The mixture is passed through filter $g(t)$. Introducing notation $h(t) = g(t) \otimes g(t)$ for the «normal» raised cosine filter with the same roll-off and taking into account that $g(t-\tau_u) \otimes g(t) = h(t-\tau_u), u = 1,2,$ we have the following output of matched filter:

$$y(t) = a_1 e^{j\varphi_1} \sum_{n=-\infty}^{\infty} s_1(n) h(t - nT_s - \tau_1) + a_2 e^{j\varphi_2} \sum_{n=-\infty}^{\infty} s_2(n) h(t - nT_s - \tau_2). \quad (1)$$

Sampling of the signal (1) at times $(\tau_1 + nT_s)$ and $(\tau_2 + nT_s)$ respectively, produces two sequences:

$$y_u(n) = a_u e^{j\varphi_u} s_u(n) + a_{u'} e^{j\varphi_{u'}} \sum_{n=-\infty}^{\infty} s_{u'}(n) h(t + \tau_1 - \tau_2) + w_u(n), u = 1,2,$$

where $u' = 3 - u$ denotes the channel index, opposite to u.

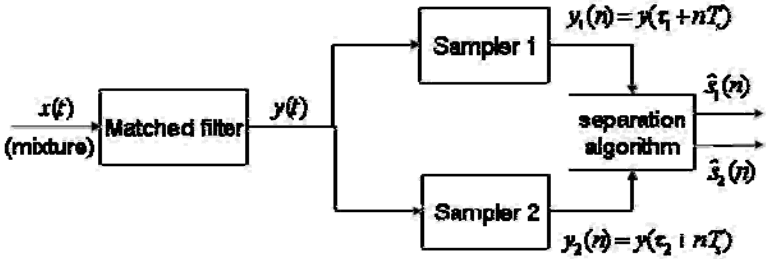

Fig. 3. *The structure of the receiver*

Let us assume that the impulse response $h(t)$ is essentially non-zero only for $(2l+1)$ symbols (a common example is $l = 2$). Using this assumption, the model of observations transforms to:

$$\begin{cases} y_1(n) = a_1 e^{j\varphi_1} s_1(n) + a_2 e^{j\varphi_2} h_{2,\tau} s_2^T(n) + w_1(n), \\ y_2(n) = a_2 e^{j\varphi_2} s_2(n) + a_1 e^{j\varphi_1} h_{1,\tau} s_1^T(n) + w_2(n), \end{cases} \quad (2)$$

where

$$h_{u,\tau} = \{h(kT_s + \tau_u - \tau_{u'}), k = -l...l,$$
$$s_u(n) = \{s_u(n+k)\}, k = -l...l.$$

So, as can be seen from observation model (2), the observed signals $y_1(n)$ and $y_2(n)$ include first signal plus weighted tail of the second and second signal plus weighted tail of the first. We assume that the observation noises w_1, w_2 possess the same variance σ^2 . So, having observations $y_1(n)$ and $y_2(n)$, our goal is to find estimates of $s_1(n)$ and $s_2(n)$.

Bayesian estimation of original QPSK sequences. The main idea of proposed approach is to maximize maximum a posteriori probability of transmitted symbols for each time instant $n = 1,\dots,N$:

$$\max_{m=1..4} P(s_u(n) = S_m / y_u(n)), u = 1,2; S_m \in \{\pm 1 \pm j\}.$$

Similarly to technique, implemented in [2], one can show that a posteriori probability for u -th signal is connected with that of the opposite signal:

$$P(s_u(n) = S_m / y_u(n)) =$$
$$= \sum_{s \in S_l} P(s_u(n) = S_m / y_u(n), s_{u'}(n) = s) P(s_{u'}(n) = s). \quad (3)$$

Assuming that the observation noise is Gaussian,

$$P(s_u(n) = S_m / y_u(n), s_{u'}(n) = s) = \exp(-0.5\sigma^{-2} d^2_{u,m,s}(n))$$

(we dropped the denominator of Gaussian density for simplicity), where a priori discrepancy is given by

$$d_{u,m,s}(n) = \left| y_u(n) - (S_m a_u \exp(j\varphi_u) + a_{u'} \exp(j\varphi_{u'}) h_{u',\tau} s^T) \right|$$

and

$$P(s_{u'}(n) = s) = \prod_{k=-l...l} P(s_{u'}(n+k) = s_{k+l+1}) .$$

Thus, formula (3) turns to

$$P(s_u(n) = S_m / y_u(n)) =$$
$$= \sum_{s \in S_l} \exp(-0.5 d_{u,m,s} \sigma^{-2}) \prod_{k=-l...l} P(s_{u'}(n+k) = s_{k+l+1}). \quad (4)$$

As can be seen, relation (4) describes interdependence of a posteriori probabilities for the opposite signals. This gives a hint to construct iterative algorithm:

$$p^{(m)}_{u,i+1}(n) = \sum_{\mathbf{s} \in \mathbf{S}_l} \exp(-0.5 d_{u,m,\mathbf{s}} \sigma^{-2}) \prod_{k=-l...l} p_{u',i}(n+k) ,$$

where i is a number of iteration. So, the iterative algorithm for the estimation of sequences $s_1(n)$ and $s_2(n)$ is as shown at Fig. 4. The iterations stop when average probabilities on adjacent iterations do not differ too much.

- 1: **Initialization**
- 2: Compute $d_{u,m,\mathrm{s}}(n)$
- 3: Initialize $p_{u,i+1}^{(m)}(n) = 1/4$
- 4: **Iterarive part**
- 5: **for** $i = 1 : maxiter$
- 6: **for** $n = 1 : N$
- 7: **for** $m = 1 : 4$
- 8: Estimate $p_{1,i+1}^{(m)}(n)$ (based on $p_{2,i}^{(m)}$ (...))
- 9: Estimate $p_{2,i+1}^{(m)}(n)$ (based on $p_{1,i}^{(m)}$ (...))
- 10: **end**
- 11: **end**
- 12: **if** $||p_{mean,i+1}^{(m)} - p_{mean,i}^{(m)}|| < threshold$
- 13: stop
- 14: **end**
- 15: **end**

Fig. 4. *The algorithm for the restoration of original QPSK sequences*

Channel parameters estimation. There is a common practice to insert predefined symbols (unique words) into the transmitted sequences. In our modeling, we use 32-symbol (64-bit) sequences denoted by U . The position of the unique word can be identified by its cross-correlation with received signal (see Fig. 5). Once we have detected the position of the unique word, we analyze its peak value $R_{i_{\max}}^{(u)}$ ($u = 1, 2$). Then the amplitude, phase and time delay can be estimated approximately as follows:

$$\hat{a}_u = R_{i_{\max}}^{(u)} / \|U\|^2 , ,$$

$$\hat{\varphi}_u = \arg R_{i_{\max}}^{(u)} ,$$

$$\hat{\tau}_u = \frac{R_{i_{\max}+1}^{(u)} - R_{i_{\max}-1}^{(u)}}{2R_{i_{\max}}^{(u)} - R_{i_{\max}+1}^{(u)} - R_{i_{\max}-1}^{(u)}}$$

The last formula for the delay comes from the parabolic interpolation of correlation function (see Fig. 6).

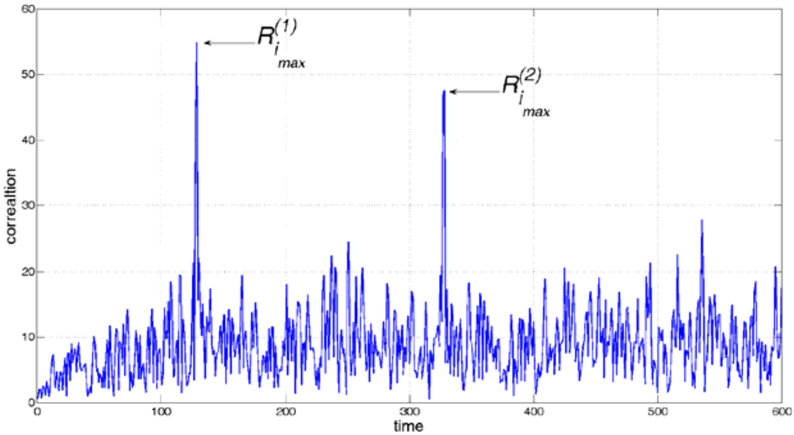

Fig. 5. *Detection of unique words in the mixture of two signals*

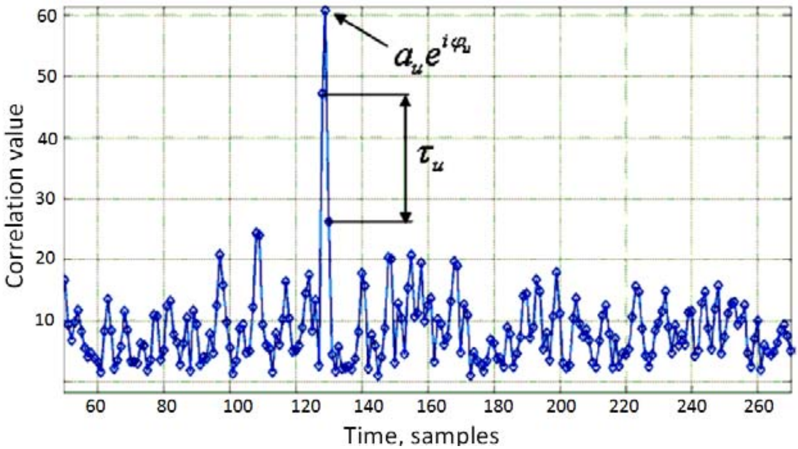

Fig. 6. *Determination of channel parameters from cross-correlation peak value*

Experimental results. In this section the performance of proposed algorithm at different signal-to-noise ratios is analyzed. We take in these experiments the following values of parameters: $a_1 = a_2 = 1$, $\varphi_1 = \varphi_2 = 0.35$. The value of time diversity $\tau = |\tau_1 - \tau_2|$ was allowed to take different values and we examined algorithm's performance for different τ.

Fig. 7 shows the performance of proposed algorithm when the channel parameters are assumed known and Fig. 8 shows the performance of proposed algorithm when the channel parameters are estimated as was discussed above.

In both cases 15 iterations of the algorithm were used. It can be seen that the case of known parameters has a slight advantage in terms of BER over the case when parameters are estimated by proposed procedure.

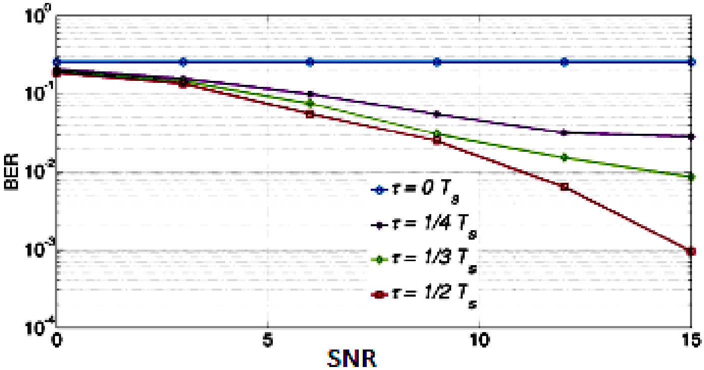

Fig. 7. *Performance of proposed algorithm when the channel parameters are assumed known*

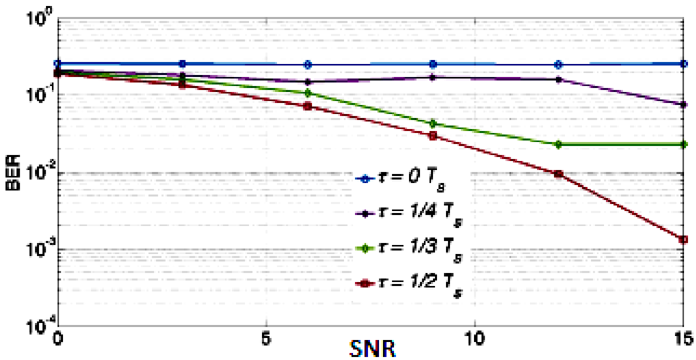

Fig. 8. *Performance of proposed algorithm when the channel parameters are estimated by proposed method*

As can be seen from the figures 7 and 8, the higher time diversity τ leads to better separation performance of the algorithm. For example, with $\tau = 0$ we have no diversity and the components of the mixture cannot be separated. On the opposite, the best performance is achieved when time diversity takes its maximum value $0.5T_s$. This shows that the algorithm may properly exploit the diversity induced by the delay between channels.

To understand better the effect of parameters estimation procedure, we consider the case $\tau = 1/3\ T_s$. Fig. 9 shows the comparison of BER for several cases:

1. Amplitudes a_1, a_2 are assumed to be known, but the phases φ_1, φ_2 and the delays τ_1, τ_2 take random values from their area of definition («partially-known» case).

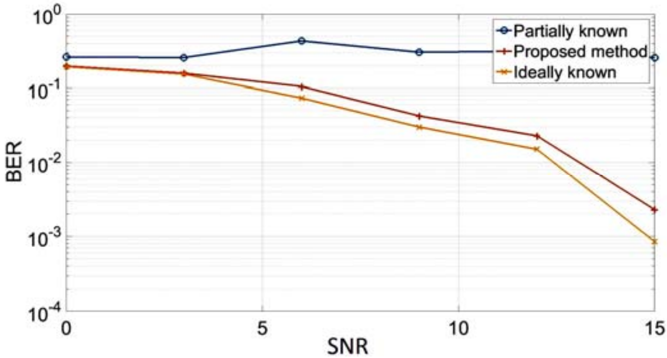

Fig. 9. *Performance of proposed parameters' estimation method for the case* $\tau = 1/3T_s$

2. All channel parameters are assumed unknown and they are estimated by the proposed procedure.
3. All channel parameters are assumed known beforehand («ideally-known» case).

From the Fig. 9 it can be seen that the proposed estimation procedure crucially improves the BER of the algorithm providing improvement over the case 1 («partially-known» parameters) from 1.3 times for $E_b / N_0 = 0$ dB to 112 times for $SNR = 15$ dB. At the same time, the ratio between proposed method estimation method and case of ideally known parameters is not large: the obtained BERs are always of the same order, the maximum ratio between them is from 1.02 times for $SNR = 0$ dB to 2.7 times for $SNR = 15$ dB. The similar conclusions are confirmed for other values of τ .

Conclusions. In this paper the new method for the single-channel separation of two QPSK signals based on iterative maximization of a posteriori probability for transmitted symbols is presented. The best performance of the method is achieved when time diversity between channels takes its maximum value, namely half of a symbol period. The essential advantage over the previously proposed approach is due to proposed procedure of channel parameters' estimation. For the case $\tau = 1/3\ T_s$ it was shown that the BER is improved from 1.3 to 112 times (for different E_b / N_0) in comparison with the case of partially known parameters. At the same time, the BER values for the proposed estimation procedure are of the same order as for the case of ideally known parameters.

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МЕТОД ІТЕРАТИВНОГО ОДНОКАНАЛЬНОГО СЛІПОГО РОЗДІЛЕННЯ QPSK-СИГНАЛІВ

Запропоновано метод одноканального сліпого розділення двох сигналів з квадратурно-фазовою маніпуляцією (QPSK). Метод базується на ітеративному оцінюванні компонентів суміші за принципом максимізації апостеріорної ймовірності. Отримані формули для відповідних апостеріорних ймовірностей та на їх основі розроблено алгоритм оцінювання компонентів суміші. Також розроблено алгоритм оцінювання параметрів каналу (амплітуд, фаз і часових затримок). Ефективність методу перевірена при різних рівнях шуму та часового рознесення між каналами. Розроблена процедура оцінювання параметрів забезпечує суттєве скорочення бітової похибки (BER) у порівнянні з випадком невідомих параметрів.

Ключові слова: *сліпе розділення, BPSK (двійкова фазова маніпуляція), QPSK (квадратурна фазова маніпуляція).*

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