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USING OF THE AOWI EFFICIENCY EVALUATION TECHNIQUE FOR RANKING OF THE REGIONAL SOCIO-ECONOMIC SYSTEMS

In this report methods of multi-criteria efficiency evaluation are developed and implemented for ranking of the socio-economic systems of the EU regions. The socio-economic ranking problem is a multi-criteria non-convex optimization problem that is solved by the implementation of a new efficiency evaluation AOWI method.

Key words: *Regional Economics, Non-Parametric Efficiency Evaluation, Welfare measure, Index Numbers and Aggregation.*

Introduction. Output/input quotients may be considered as a business ratio for the evaluation of efficiency. At first glance (at least), a higher value provides better achievement than a lower value. If, however, one regards two such characteristics, then it is no longer so simple to determine the advantages clearly. The situation will become more complicated, the more output and input exist in the model. This problem of efficiency measurement in cases of several dimensions was examined to a large extent in recent years in management economics and a class of methods known as Data Envelopment analysis (DEA) [5-7] was developed for their evaluation.

But sometimes an application of DEA is problematic. For example, the values of indicators used in DEA can't be stochastic, Boolean or negative and the used units must be as homogeneous as possible. To overcome the problem of stochastic outputs and/or inputs in [2] DEA was extended to cases with geometrical or stochastic uncertainties of data. So it was possible to define the efficiency and effectiveness in cases when we do not know the values of the indicators exactly, but only know a polytope where they lay or – in stochastic case – a distribution of indicators. In these cases efficiency is no longer a number, but an interval. To find this interval we have used the methods of linear programming and of minimax optimization.

In [13] SOW-Index was proposed to overcome difficulties with the non-negativity of indicators and with the homogeneity of units. This method calculates the absolute distance to the efficiency border.

In this paper we modify SOW-Index and use it for the measurement of the social welfare and for the comparison of social systems in different regions. The indicator weights are calculated as solutions of the explicit absolute optimal weights index (AOWI) procedure and are endogenously optimally selected for each region. The solution of this complicated, nonlinear and non-convex problem reflects the development of the social systems of individual regions adequately representing each region independent of subjective priorities. Using the proposed wealth measure, we compute and evaluate the uniform ranking of the regions within the European Union in the social framework.

The benchmarking of the social welfare has a strong influence on political decisions of the European Commission and is an important topic concerning the broad discussion about validation, reasons, and sequences of the future extension of the European Union. The critical point is that the differences within the European Union are large and the entry of new countries will make the inequalities of the European Union regions more strongly expressed.

Multi-objective efficiency evaluation. The advantage of the multi-objective criteria concept of efficiency evaluation in the economic science lies in the possibility to investigate systems that cannot be captured by only one measurement. The multi-objective problem with maximization of n outputs can be defined as the problem of the simultaneous optimization of n functions:

$$y_1 = F_1(x), y_2 = F_2(x), \ldots, y_n = F_n(x). \qquad (1)$$

The scalar concept of “optimality" does not apply directly in the multi-criteria setting. A useful notion here is Pareto optimality. A point $x^{opt} \in X$ is said to be Pareto optimal if, and only if, there is no $x \in X$ such that $F_i(x) \geq F_i(x^{opt})$ for all $i = 1,\ldots,n$ with at least one strict inequality.

Often instead of (1) one solves a single-criterion problem by combining the multiple criteria into one scalar function F with non-negative weights $u_i \geq 0$, $\sum_i u_i > 0$

$$F(x) = \sum_i u_i F_i(x). \qquad (2)$$

The maximizer of this combined function is Pareto optimal. It is up to the user to choose appropriate weights. Variation of the convex weights generates various points in the Pareto set.

The Pareto optimal solution for many real-life multi-criteria problems can also be found by Data Envelopment Analysis. DEA considers the given data (x_j, y_j) of the *j*-th production unit, for $j = 1,2,\ldots,n$ with input vectors $x_j \geq 0$ and output vectors $y_j \geq 0$.

For each unit one can form the virtual input $v'x_j$ and virtual output $u'y_j$ with weights $v \geq 0$ and $u \geq 0$. Then one tries to determine the weights that maximize the ratio “virtual output” vs. “virtual input”. The optimal weights of the *j*-th unit are calculated as the solution of the following mathematical programming problem:

$$\max_{u \geq 0, v \geq 0} \left(u'y_i / v'x_i \right) : u'y_j / v'x_j \leq 1, \; j = 1,2,...,n.$$

The constraints mean that the ratio of “virtual output” vs. “virtual input” should not exceed 1 for any unit. Uniqueness of the solution is achieved by imposing the constraint $v'x_i$, i.e.:

$$\max_{u \geq 0, v \geq 0} u'y_i : u'y_j - v'x_j \leq 1, \; v'x_i = 1, \; j = 1,2,...,n.$$

Using the duality in linear programming, one can derive an equivalent input-orientated envelopment form of this problem:

$$\min_{\theta, \lambda \geq 0} \theta : \theta x_i - \sum_{j=1}^{n} x_j \lambda_j \geq 0, \; \sum_{j=1}^{n} x_j \lambda_j - y_i \geq 0,$$

where θ is scalar and λ is a nx_1 vector.

However, the piece-wise linear form of the non-parametric frontier in DEA can cause difficulties in efficiency measurement because of the sections of the piece-wise linear frontier that may run parallel to the axes. In this case one can reduce the amount of input used and still produce the same output (this is known as input slack in the literature); or one can increase the amount of output produced and still use the same input (this is known as output slack). Therefore, the efficiencies θ are usually evaluated as solutions of the following input-oriented LP model (in the LP we minimize the multiple θ of inputs required to produce at least its outputs, minus a small multiple ε of the sum of input slacks s^+ and output slacks s^-):

$$\min_{\theta, \lambda \geq 0, s^+ \geq 0, s^- \geq 0} \theta - \varepsilon \left(e'^+ s^+ + e'^- s^- \right) : \sum_{j=1}^{n} x_j \lambda_j + s^+ = \theta x_i, \sum_{j=1}^{n} y_j \lambda_j + s^- = y_i \quad (3)$$

e^+, e^- are vectors of ones.

DEA is now a broadly used method for efficiency estimation. In [10] DEA and Free Disposable Hull (FDH) were used to evaluate the efficiency of the health and education systems in 140 developing countries. In contrast to DEA the FDH assumes free disposability of resources. In addition to DEA, in [1] the semi-parametric DEA/Tobit two-step procedure was applied. DEA was also used in [14] to assess the relative efficiency and flexibility of public spending on health care, education, and social protection in Slovenia compared to the advanced and new EU member states. In [12] DEA was applied to analyze the efficiency of 72 public German universities for the years 1998-2003. The DEA-score method was also applied to the practical problem of efficiency evaluation of housing promotion in Austrian regions [2].

The two new BEOW (Border-oriented Equal Optimal Weights)-Index and the EEOW (Envelope-oriented Equal Optimal Weights)-Index were applied in [9] for the benchmarking of location attractiveness of different regions representing each region in the best possible way. The designed benchmarking was important for firms making decisions on firm settlements. We also mention here the application [4].

Absolute Optimal Weights Index (AOWI). The new method of multi-criteria was proposed in [13] for the case of units with no input and multi-dimensional output y_j, $j = 1,...,n$. For the evaluation of the systems' efficiency and the corresponding ranking we will use the Absolute Optimal Weights Index (AOWI)-procedure:

$$\min_{\theta,u} \theta : \theta = \max_i \left(u' y_i\right) - u' y_j, \; Au \geq 0, a \leq u \leq b \quad (4)$$

with fixed matrix A and vectors a and b.

The value θ can be obtained by solving a non-convex multi-objective problem. For this purpose we developed and implemented a special algorithm.

The main difference between the DEA-efficiency (3) and the AOWI-efficiency (4) consists in the stronger economic contents of the AOWI-efficiency. The DEA-efficiency computes the relative normalized distance to the efficient frontier. Thus, it contains no adequate information about the absolute "economic" values of outputs. By contrast, the AOWI-efficiency defines the real space distance of the values of a unit's output to the efficiency frontier and thus the AOW-Index gives more useful information about the economic efficiency.

The second advantage of the AOWI is based on the easiness of its implementation, because the indicators in the AOW-model can be negative. This issue may be important, because many economic indicators may have negative values. The economic content of the indicators involved in the computation of the AOW is not destroyed. The implementation of the AOW needs no transformations of the original data.

The constraints on weights in (4) ensure that no weight can be 0 or 1. So we can overcome the boundary problem in the DEA that diagnoses any unit supporting the frontier to be well performing even if it performs well with respect to a single output and poorly with respect to all the others. In an extreme case, the DEA selects a weight of 1 for this output and a weight of 0 for the other outputs. Only the output with a weight of 1 is taken into account for computing the performance index.

Social framework. In this paper we used a benchmarking model in order to define the efficiency of social systems of the EU-27 member states regions. With this method we can compare resource availability and the quality of social systems throughout the EU countries. The model is based on the set of multiple statistical data about economical and social development of the regions. The set of indicators also retain such macroeconomic variables of development as GDP. Economic growth, social security, and political adjustments to the labor market can bring positive reciprocal effects in social and economic frameworks. The social dimension works as a productive factor. For example, a good state of health of the population contributes to good economic performance.

The increase of the participation rates, job market involvement of disadvantaged groups, the avoidance of social exclusion and poverty can improve economic performance. The social reforms presented in government programs of all countries take into account the aims of budget consolidation, the change of work and ways of life, and above all the forthcoming demographic development. These reforms intended the preservation of three principal purposes of the social security system: the safekeeping of an appropriate and socially fair proficiency level, the ability to finance the needs of a social system, and the adaptation according to changing needs of society and people. A further aim is to guarantee a support for socially disadvantaged people in order to make their economic, social, and cultural (re-)integration into our society possible. The social security is a fixed component of the employment system. Among others, social risks comprise age, retirement, age-conditioned dependence, death of the breadwinner, disability, illness, maternity, childcare, unemployment, and sometimes also the support of older, handicapped, or ill members of the family.

The political system can hardly determine the priorities among the social needs. However, the priorities have to be admitted. In this publication we will determine the priorities within the social needs for each region. These priorities differ from region to region and are determined through a logical procedure, which is free of the subjective priorities of the experts and depends only on the configuration of all other regions. Based on this procedure one may give recommendations how to change the region's social structure, which directions primarily have to be developed such that the positioning of the region in the reference to all other European Union regions will be improved most effectively.

Welfare indicators. We have chosen 16 important indicators in social framework, which will serve as a basis for the benchmarking of regions. These indicators can be divided into the following subgroups:

- regional gross domestic product (GDP in PPS per inhabitant),
- population (aging of population),
- education (lifelong learning, secondary and tertiary education levels of employees),
- labor force (employment rate, woman employment rate, unemployment),
- household accounts (disposable income in PPCS per inhabitant, inequality of incomes),
- differences between women and men (women and men in decision positions, earned incomes of women and men),
- health and security (female and male life expectancy and accidents),
- social cohesion.

All indicators of welfare can be partitioned into two categories “stimulus” and “detriment”, depending on whether these indicators have positive or negative effect on welfare. All indicators have to be aligned positively, i.e. the higher the value of an indicator is the better the welfare situation is. The data for the “stimuli” remained as given, the data for the “detriments” were multiplied with “-1”. The data were then linearly transformed into the interval [0.1].

The AOWI model has 12 “stimuli” (S) and 4 “detriments” (D). 10 of these variables are regional and are defined on the NUTS 2-level (R). 6 other variables are aggregate and defined on the countries’ level (A). *Table 1* gives the name, the category, and the availability of used indicators.

Table 1

The indicators used in AOWI model

Nr.	Indicator	Category	Availability
1	GDP	S	R
2	lifelong learning	S	R
3	employed persons with the second level of education	S	R
4	employed persons with the third level of education	S	R
5	employment rate	S	R
6	woman employment rate	S	R
7	disposable income of private households	S	R
8	females in parliament	S	A
9	ratio of female to male earned income	S	A
10	female life expectancy	S	A
11	male life expectancy	S	A
12	social cohesion	S	A
13	old age dependency	D	R
14	unemployment rate	D	R
15	inequality in income	D	A
16	accidents	D	R

Results of the computations and welfare implications. As the result of the implementation of a new evaluation technique AOWI we have obtained a ranking in the social framework of 268 NUTS 2-regions from the 27 European Union countries. The benchmarking shows us which regions are strongly developed in social framework and which are not.

The top 10 regions are Stockholm, Inner London, the Finnish regions Etelä-Suomi and Åland; the Swedish regions Västsverige, Sydsverige, Östra Mellansverige, and Småland med öarna; the English regions Berkshire, Bucks and Oxfordshire; and Denmark. The worst regions are the Romanian regions Southeast, Southwest, Northeast, West, South, Northwest; the Bulgarian region Severozapaden; the Polish region Podlaskie; and the Portuguese region Alentejo.

Table 2 displays the 10 top and *table 3* the 10 worst regions in the ranking and their distance to the efficiency border.

Table 2

The 10 top European Union regions in social framework

Ranking	NUTS 2	Region	Distance to the efficiency border
1	SE01	Stockholm	0.0000
2	UKI1	Inner London	0.0118
3	FI18	Helsinki (Etelä-Suomi)	0.0747
4	FI20	Åland	0.0972
5	SE0A	Västsverige	0.1007
6	SE04	Sydsverige	0.1071
7	SE02	Östra Mellansverige	0.1137
8	SE09	Småland med öarna	0.1162
9	UKJ1	Berkshire, Buckinghamshire and Oxfordshire	0.1417
10	DK00	Danmark	0.1428

Table 3

The 10 worst EU regions in social framework

Ranking	NUTS 2	Region	Distance to the efficiency border
259	RO07	Centru	1.3181
260	PT18	Alentejo	1.3242
261	PL34	Podlaskie	1.3511
262	BG11	Severozapaden	1.3576
263	RO06	Nord-Vest	1.3710
264	RO03	Sud	1.3782
265	RO05	Vest	1.3957
266	RO01	Nord-Est	1.4173
267	RO04	Sud-Vest	1.4441
268	RO02	Sud-Est	1.4610

With the help of AOWI one can determine the strong sides of the top regions as well as the reasons for the underdevelopment of the weak regions. Thus, all indicators were responsible in equal shares for the top position of Stockholm. The region Inner London was positioned well because of good values for GDP, lifelong learning, employed persons with the third level of education, disposable income of private households, and the small values for old age dependency, and accidents. The weak placement of the Romanian regions Southeast and Southwest at the bottom of the benchmarking is caused by bad values for lifelong learning, employed persons with the third level of education, female employment rate, disposable income of private households, females in parliament, ratio of female to male earned income, female and male life expectancy, and social cohesion.

The results of the benchmarking attest the great social divergence between the old European regions and the new ones – Bulgarian and Rumanian regions. The best position within the regions has the Bulgarian region Yugozapaden (211). All other Bulgarian and Rumanian regions lie on the lower end of the ranking. In sum the Bulgarian regions do better than Rumanian ones.

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В даній роботі будуються і використовуються методи багатокритеріального ранжування соціально-економічних систем в різних регіонах ЄС. Задача соціально-економічного ранжування є задачею невипуклої багатокритеріальної оптимізації, яка розв’язується запропонованим методом оцінювання AOWI.

Ключові слова: *регіональна економіка, непараметричне оцінювання ефективності, оцінка добробуту, індекси показників і агрегація.*

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