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The mathematical model of process of aerobic sewage treatment in the porous environment which considers interaction of bacteria, organic and biologically not oxidising substances is constructed. The offered algorithm of the decision corresponding modelling nonlinear singular the indignant problem of type «convection-diffusion-mass exchange» with time delay

Key words: *nonlinear problems, process of aerobic clearing, filtering, singular indignations, asymptotic, time delay.*

Отримано: 06.03.2011

УДК 517.9+531.19+530.145

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ON EVOLUTION EQUATIONSFOR MARGINAL CORRELATION OPERATORS

This paper is devoted to the problem of the description of nonequilibrium correlations of quantum many-particle systems. A non-perturbative solution of the Cauchy problem of the nonlinear quantum BBGKY hierarchy for marginal correlation operators is constructed as an expansion over particle clusters which evolution is governed by the corresponding-order cumulant of the nonlinear groups of operators generated by the von Neumann hierarchy.

Key words: *nonlinear quantum BBGKY hierarchy, von Neumann hierarchy, correlation operator, quantum many-particle system.*

Introduction. The importance of the mathematical description of correlations in numerous problems of the modern statistical mechanics is well-known. Among them in particular, we refer to such fundamental problem as the problem of a description of collective behavior of interacting particles by quantum kinetic equations [1—8]. Owing to the intrinsic complexity and richness of these problems, primarily it is necessary to develop an adequate mathematical theory of underlying evolution equations.The goal of the paper is to derive rigorously the evolution equations for marginal correlation operators that give an equivalent approach to the description of the evolution of states in comparison with marginal density operators governed by the quantum BBGKY hierarchy and to construct a solution of the corresponding Cauchy problem.

The von Neumann hierarchy. We consider a quantum system of a non-fixed, i.e. arbitrary but finite, number of identical (spinless) particles with unit mass $m=1$ in the space $\mathbb{R}^{\nu}, \nu \geq 1$, that obey the Maxwell-Boltzmann statistics. Let $\mathcal{F}_{\mathcal{H}} = \bigoplus_{n=0}^{\infty} \mathcal{H}_n$ be the Fock space over the Hilbert space $\mathcal{H}$, where the n-particle Hilbert space $\mathcal{H}_n \equiv \mathcal{H}^{\otimes n}$ is a tensor product of n Hilbert spaces $\mathcal{H}$ and we adopt the usual convention that $\mathcal{H}_0 = \mathbb{C}$. The Hamiltonian H_n of the n-particle system is a self-adjoint operator with the domain $\mathcal{D}(H_n) \subset \mathcal{H}_n$

$$H_n = \sum_{i=1}^{n} K(i) + \sum_{i_1 < i_2 = 1}^{n} \Phi(i_1, i_2), \tag{1}$$

where $K(i)$ is the operator of a kinetic energy of the i particle and $\Phi(i_1, i_2)$ is the operator of a two-body interaction potential. In particular on functions ψ_n that belong to the subspace $L_0^2(\mathbb{R}^{\nu n}) \subset \mathcal{D}(H_n) \subset L^2(\mathbb{R}^{\nu n})$ of infinitely differentiable symmetric functions with compact supports the operator $K(i)$ acts according to the formula: $K(i)\psi_n = -\frac{\hbar^2}{2}\Delta_{q_i}\psi_n$, where $2\pi\hbar$ is a Planck constant, and for the operator Φ we have: $\Phi(i_1, i_2)\psi_n = \Phi(q_{i_1}, q_{i_2})\psi_n$, respectively. We assume that the function $\Phi(q_{i_1}, q_{i_2})$ is symmetric with respect to permutations of arguments and it is translation-invariant bounded function.

States of a system of the Maxwell-Boltzmann particles belong to the space $\mathfrak{L}^1(\mathcal{F}_{\mathcal{H}}) = \bigoplus_{n=0}^{\infty} \mathfrak{L}^1(\mathcal{H}_n)$ of sequences $f = (f_0, f_1, \ldots, f_n, \ldots)$ of trace-class operators $f_n \equiv f_n(1, \ldots, n) \in \mathfrak{L}^1(\mathcal{H}_n)$ and $f_0 \in \mathbb{C}$, that satisfy the symmetry condition: $f_n(1,\ldots,n) = f_n(i_1,\ldots,i_n)$ for arbitrary $(i_1,\ldots,i_n) \in (1,\ldots,n)$, equipped with the norm

$$\|f\|_{\mathfrak{L}^1(\mathcal{F}_{\mathcal{H}})} = \sum_{n=0}^{\infty} \|f_n\|_{\mathfrak{L}^1(\mathcal{H}_n)} = \sum_{n=0}^{\infty} \boldsymbol{Tr}_{1,\ldots,n} \left| f_n(1,\ldots,n) \right|,$$

where $\boldsymbol{Tr}_{1,\ldots,n}$ are partial traces over $1,\ldots,n$ particles [12]. We denote by $\mathfrak{L}_0^1(\mathcal{F}_{\mathcal{H}})$ the everywhere dense set in $\mathfrak{L}^1(\mathcal{F}_{\mathcal{H}})$ of finite sequences of degenerate operators with infinitely differentiable kernels with compact supports.

We describe states of a system by means of sequences $g(t) = (g_0, g_1(t,1),\ldots,g_n(t,1,\ldots,n),\ldots) \in \mathfrak{L}^1(\mathcal{F}_{\mathcal{H}})$ of the correlation operators $g_n(t)$, $n \geq 1$. The evolution of all possible states is determined by the initial-value problem of the von Neumann hierarchy

$$\frac{d}{dt} g_s(t,Y) = \mathcal{N}\left(Y \middle| g(t)\right), \tag{2}$$

$$g_s(t,Y)\Big|_{t=0} = g_s(0,Y), \; s \geq 1, \tag{3}$$

where the following notations are used:

$$\begin{aligned} \mathcal{N}\left(Y \middle| g(t)\right) &= \mathcal{N}_s(Y) g_s(t,Y) + \\ &+ \sum\nolimits_{\boldsymbol{P}:\, Y = X_1 \bigcup X_2} \sum\nolimits_{i_1 \in X_1} \sum\nolimits_{i_2 \in X_2} \left(-\mathcal{N}_{\mathrm{int}}(i_1,i_2)\right) g_{|X_1|}(t,X_1) g_{|X_2|}(t,X_2), \end{aligned} \tag{4}$$

$\sum_{\boldsymbol{P}:\, Y = X_1 \bigcup X_2}$ is the sum over all possible partitions $\boldsymbol{P}$ of the set $Y \equiv (1,\ldots,s)$ into two nonempty mutually disjoint subsets $X_1 \subset Y$ and $X_2 \subset Y$, the operator $(-\mathcal{N}_s)$ defined on $\mathfrak{L}_0^1(\mathcal{H}_s)$ by the formula

$$\left(-\mathcal{N}_s(Y)\right) = -\frac{i}{\hbar}\left(H_s f_s - f_s H_s\right), \tag{5}$$

is the generator of the von Neumann equation [13] and the operator $(-\mathcal{N}_{\mathrm{int}})$ is defined by

$$\left(-\mathcal{N}_{\mathrm{int}}(i_1,i_2)\right) f_s = \frac{i}{\hbar}\left(\Phi(i_1,i_2) f_s - f_s \Phi(i_1,i_2)\right). \tag{6}$$

Hereafter we use the following notations: $(\{X_1\},\ldots,\{X_{|P|}\})$ is a set, elements of which are $|P|$ mutually disjoint subsets $X_i \subset Y \equiv (1,\ldots,s)$ of the partition $\boldsymbol{P}: Y = \bigcup_{i=1}^{|P|} X_i$, i.e. $\left|(\{X_1\},\ldots,\{X_{|P|}\})\right| = |P|$. In view of these notations we state that $(\{Y\})$ is the set consisting of one element

$Y=(1,\dots,s)$ of the partition $\boldsymbol{P}\left(|P|=1\right)$ and $\left|\left(\{Y\}\right)\right|=1$. We introduce the declasterization mapping $\theta:\left(\{X_1\},\dots,\{X_{|P|}\}\right)\rightarrow Y$, by the following formula: $\theta:\left(\{X_1\},\dots,\{X_{|P|}\}\right)=Y$. For example, let $X\equiv(1,\dots,s+n)$, then for the set $\left(\{Y\},X\setminus Y\right)$ it holds: $\theta\left(\{Y\},X\setminus Y\right)=X$.

On the space $\mathfrak{L}^1(\mathcal{H}_n)$ we also introduce the mapping: $\mathbb{R}\ni t\rightarrow\mathcal{G}_n(-t)f_n$, which is generated by the solution of the von Neumann equation of n particles [13]

$$\mathcal{G}_n(-t)f_n=e^{-\frac{i}{\hbar}tH_n}f_n e^{\frac{i}{\hbar}tH_n}. \tag{7}$$

This mapping is an isometric strongly continuous group that preserves positivity and self-adjointness of operators [12]. On $\mathfrak{L}_0^1(\mathcal{H}_n)\subset\mathcal{D}(-\mathcal{N}_n)$ the infinitesimal generator of group (7) is determined by operator (5).

A solution of the Cauchy problem (2)—(3) is given by the following expansion [9—10]

$$\begin{aligned}g_s(t,Y)&=\mathcal{G}\left(t;Y\,\middle|\,g(0)\right)=\\&=\sum_{\boldsymbol{P}:\,Y=\bigcup_i X_i}\mathfrak{A}_{|\boldsymbol{P}|}\left(t,\{X_1\},\dots,\{X_{|\boldsymbol{P}|}\}\right)\prod_{X_i\subset\boldsymbol{P}}g_{|X_i|}(0,X_i),\ s\geq 1,\end{aligned} \tag{8}$$

where $\sum_{\boldsymbol{P}:\,Y=\bigcup_i X_i}$ is the sum over all possible partitions $\boldsymbol{P}$ of the set $Y\equiv(1,\dots,s)$ into $|P|$ nonempty mutually disjoint subsets $X_i\subset Y$, the evolution operator $\mathfrak{A}_{|P|}(t)$ is the $|P|\,th$-order cumulant of groups of operators (7) defined by the formula

$$\begin{aligned}\mathfrak{A}_{|\boldsymbol{P}|}\left(t,\{X_1\},\dots,\{X_{|\boldsymbol{P}|}\}\right)=&\sum_{\boldsymbol{P}':\left(\{X_1\},\dots,\{X_{|P|}\}\right)=\bigcup_k Z_k}(-1)^{|\boldsymbol{P}'|-1}\times\\&\times\left(|\boldsymbol{P}'|-1\right)!\prod_{Z_k\subset\boldsymbol{P}'}\mathcal{G}_{|\theta(Z_k)|}\left(-t,\theta(Z_k)\right)\end{aligned} \tag{9}$$

Here $\sum_{\boldsymbol{P}':\left(\{X_1\},\dots,\{X_{|P|}\}\right)=\bigcup_k Z_k}$ is the sum over all possible partitions $\boldsymbol{P}'$ of the set $\left(\{X_1\},\dots,\{X_{|P|}\}\right)$ into $|\boldsymbol{P}'|$ nonempty mutually disjoint subsets $Z_k\subset\left(\{X_1\},\dots,\{X_{|P|}\}\right)$. For operators (8) the estimate holds

$$\left\| g_s(t) \right\|_{\mathfrak{L}^1(\mathcal{H}_s)} < s! e^{3s} \boldsymbol{c}^s, \tag{10}$$

where $\boldsymbol{c} \equiv \max\limits_{P:\, Y=\bigcup_i X_i} \left(\left\| g_{|X_1|}(0) \right\|_{\mathfrak{L}^1(\mathcal{H}_{|X_1|})}, \ldots, \left\| g_{\left| X_{|P|} \right|}(0) \right\|_{\mathfrak{L}^1\left(\mathcal{H}_{\left|X_{|P|}\right|}\right)} \right)$.

If $g_n(0) \in \mathfrak{L}_0^1(\mathcal{H}_n) \subset \mathfrak{L}^1(\mathcal{H}_n),\, n \geq 1$, expansion (8) is a strong (classical) solution of the Cauchy problem (2)—(3) and for arbitrary initial data $g_n(0) \in \mathfrak{L}^1(\mathcal{H}_n),\, n \geq 1$, it is a weak (generalized) solution.

In case of the absence of correlations between particles at initial time, i.e. the initial data satisfying the a chaos condition, the sequence of correlation operators has the form

$$g(0) = \left(0, g_1(0,1), 0, \ldots\right). \tag{11}$$

The corresponding solution of the initial-value problem of the von Neumann hierarchy is given by the expansion

$$g_s(t,Y) = \mathfrak{A}_s(t,Y) \prod_{i=1}^{s} g_1(0,i), \tag{12}$$

where $\mathfrak{A}_s(t)$ is the sth -order cumulant defined by

$$\mathfrak{A}_s(t,Y) = \sum_{P:\, Y=\bigcup_i X_i} (-1)^{|P|-1} (|P|-1)! \prod_{X_i \subset P} \mathcal{G}_{|X_i|}(-t, X_i). \tag{13}$$

We note that correlations created in evolutionary process of a system are described by formula (12) and determined by the corresponding-order cumulant of the groups of operators (7) of the von Neumann equations.

Marginal correlation operators and marginal density operators. We introduce the marginal correlation operators by the series

$$G_s(t,1,\ldots,s) = \sum_{n=0}^{\infty} \frac{\boldsymbol{1}}{n!} \boldsymbol{Tr}_{s+1,\ldots,s+n} g_{s+n}(t,1,\ldots,s+n),\; s \geq 1, \tag{14}$$

where the sequence $g_{s+n}(t,1,\ldots,s+n),\, n \geq 0$, is a solution of the Cauchy problem of the von Neumann hierarchy (2).

Traditionally marginal correlation operators are introduced by means of the cluster expansions of the marginal density operators $F_s(t),\, s \geq 1$, governed by the quantum BBGKY hierarchy [11]

$$F_s(t,Y) = \sum_{P:Y=\bigcup_i X_i} \prod_{X_i \subset P} G_{|X_i|}(t, X_i),\; s \geq 1, \tag{15}$$

where $\sum_{P:Y=\bigcup_i X_i}$ is the sum over all possible partitions P of the set $Y\equiv(1,\ldots,s)$ into $|P|$ nonempty mutually disjoint subsets $X_i\subset Y$. Hereupon solutions of cluster expansions (15)

$$G_s\left(t,Y\right)=\sum_{P:Y=\bigcup_i X_i}\left(-1\right)^{|P|-1}\left(|P|-1\right)!\prod_{X_i\subset P}F_{|X_i|}\left(t,X_i\right), \tag{16}$$

are interpreted as the operators that describe correlations of many-particle systems. Thus, marginal correlation operators (16) are cumulants (semi-invariants) of the marginal density operators.

The marginal (s-particle) density operators (15) are determined by the Cauchy problem of the quantum BBGKY hierarchy [11]

$$\frac{d}{dt}F_s\left(t,Y\right)=-\mathcal{N}_s\left(Y\right)F_s\left(t,Y\right)+\sum_{i\in Y}\boldsymbol{Tr}_{s+1}\left(-\mathcal{N}_{\mathrm{int}}\left(i,s+1\right)\right)F_{s+1}\left(t\right), \tag{17}$$

$$F_s\left(t\right)\Big|_{t=0}=F_s\left(0\right),\ s\geq 1. \tag{18}$$

If $F\left(0\right)\in\mathfrak{L}^1_\alpha\left(\mathcal{F}_{\mathcal{H}}\right)=\bigoplus_{n=0}^{\infty}\alpha^n\mathfrak{L}^1_\alpha\left(\mathcal{H}_n\right)$ and $\alpha>e$, then for $t\in\mathbb{R}$ a unique solution of the Cauchy problem (17)—(18) of the quantum BBGKY hierarchy exists and is given by the expansion [9; 15]

$$F_s\left(t,Y\right)=\sum_{n=0}^{\infty}\frac{1}{n!}\boldsymbol{Tr}_{s+1,\ldots,s+n}\mathfrak{A}_{1+n}\left(t,\{Y\},X\setminus Y\right)F_{s+n}\left(0,X\right),\ s\geq 1, \tag{19}$$

where the $\left(1+n\right)th$-order cumulant $\mathfrak{A}_{1+n}\left(t\right)$ of groups of operators (7) is defined by

$$\begin{aligned}\mathfrak{A}_{1+n}\left(t,\{Y\},X\setminus Y\right)=&\sum_{P:\left(\{Y\},\,X\setminus Y\right)=\bigcup_i X_i}\left(-1\right)^{|\boldsymbol{P}|-1}\left(|\boldsymbol{P}|-1\right)!\ \times\\&\times\prod_{X_i\subset\boldsymbol{P}}\mathcal{G}_{|\theta(X_i)|}\left(-t,\theta\left(X_i\right)\right),\end{aligned} \tag{20}$$

where $\sum_P$ is the sum over all possible partitions P of the set $\left(\{Y\},\,X\setminus Y\right)$ into $|P|$ nonempty mutually disjoint subsets $X_i\subset\left(\{Y\},\,X\setminus Y\right)$.

Formally, the evolution equations for marginal correlation operators are derived from the quantum BBGKY hierarchy for marginal density operators (15) on basis of expression (16). Then the evolution of all possible states of quantum many-particle systems obeying the Maxwell-Boltzmann statistics with the Hamiltonian (1) can be described within the framework of marginal correlation operators governed by the nonlinear quantum BBGKY hierarchy

$$\frac{d}{dt}G_s\left(t,Y\right)=\mathcal{N}\left(Y\middle|G\left(t\right)\right)+\boldsymbol{Tr}_{s+1}\sum_{i\in Y}\left(-\mathcal{N}_{\mathrm{int}}\left(i,s+1\right)\right)\times$$

$$\times(G_{s+1}(t,Y,s+1)+\sum_{\substack{P:(Y,s+1)=X_1\bigcup X_2,\\ i\in X_1;s+1\in X_2}} G_{|X_1|}(t,X_1)\,G_{|X_2|}(t,X_2)), \tag{21}$$

$$G_s(t,Y)\Big|_{t=0}=G_s(0,Y),\ s\geq 1. \tag{22}$$

In equation (21) the operator $\mathcal{N}(Y\,|\,G(t))$ is generator of the von Neumann hierarchy (2) defined by formula (4), i.e.

$$\begin{aligned}\mathcal{N}\big(Y\big|G(t)\big)&=\big(-\mathcal{N}_s(Y)\big)G_s(t,Y)+\\ &+\sum_{P:\,Y=X_1\bigcup X_2}\ \sum_{i_1\in X_1}\ \sum_{i_2\in X_2}\big(-\mathcal{N}_{\mathrm{int}}(i_1,i_2)\big)G_{|X_1|}(t,X_1)\,G_{|X_2|}(t,X_2)\end{aligned} \tag{23}$$

where the operators $(-\mathcal{N}_s)$ and $(-\mathcal{N}_{\mathrm{int}})$ are defined by (5) and (6) respectively, $\sum\limits_{P:\,Y=X_1\bigcup X_2}$ is the sum over all possible partitions P of the set $Y\equiv(1,\ldots,s)$ into two nonempty mutually disjoint subsets $X_1\subset Y$ and $X_2\subset Y$, and $\sum\limits_{\substack{P:(Y,s+1)=X_1\bigcup X_2,\\ i\in X_1;s+1\in X_2}}$ is the sum over all possible partitions of the set $(Y,s+1)$ into two mutually disjoint subsets X_1 and X_2 such that ith particle belongs to the subset X_1 and $(s+1)th$ particle belongs to X_2. As far as we know hierarchy (21) was introduced by M. M. Bogolyubov [11] and in the papers of J. Yvon [16] and M. S. Green [17] for systems of classical particles.

Another method of the justification of evolution equations for marginal correlation operators consists in their derivation from the von Neumann hierarchy for correlation operators (2) on basis of definition (14).

We emphasize that the evolution of marginal correlation operators of both finitely and infinitely many quantum particles is described by initial-value problem of the nonlinear quantum BBGKY hierarchy (21). For finitely many particles the nonlinear quantum BBGKY hierarchy is equivalent to the von Neumann hierarchy (2).

A non-perturbative solution of the nonlinear quantum BBGKY hierarchy. To construct a non-perturbative solution of the Cauchy problem (21)—(22) of the nonlinear quantum BBGKY hierarchy we first consider its structure for physically motivated example of initial data, namely, initial data satisfying a chaos property

$$G_s(t,Y)\Big|_{t=0}=G_1(0,1)\,\delta_{s,1},\ s\geq 1, \tag{24}$$

where $\delta_{s,1}$ is a Kronecker symbol. Chaos property (24) means the absence of state correlations in a system at the initial time.

According to definition (14) and solution expansion (12), in the case under consideration the following relation between the marginal correlation operators and correlation operators is true

$$G_1(0,i) = g_1(0,i). \tag{25}$$

Taking into account the form (12) of a solution of the initial-value problem of the von Neumann hierarchy (2) in case of initial data (11), for expansion (14) we obtain

$$G_s(t,Y) = \sum_{n=0}^{\infty} \frac{1}{n!} \mathbf{Tr}_{s+1,\ldots,s+n} \mathfrak{A}_{s+n}(t,1,\ldots,s+n) \prod_{i=1}^{s+n} g_1(0,i), \tag{26}$$

where $\mathfrak{A}_{s+n}(t)$ is the $(s+n)th$ -order cumulant (13). In consequence of relation (25) we finally derive

$$G_s(t,Y) = \sum_{n=0}^{\infty} \frac{1}{n!} \mathbf{Tr}_{s+1,\ldots,s+n} \mathfrak{A}_{s+n}(t,X) \prod_{i=1}^{s+n} G_1(0,i). \tag{27}$$

If $\| G_1(0) \|_{\mathfrak{L}^1(\mathcal{H})} \leq (2e)^{-1}$, series (27) converges, since for cumulants (13) the estimate holds [9]

$$\| \mathfrak{A}_n(t) f \|_{\mathfrak{L}^1(\mathcal{H}_n)} \leq n!\, e^n \| f \|_{\mathfrak{L}^1(\mathcal{H}_n)}.$$

From the structure of series (27) it is clear that in case of absence of correlations at initial instant in a system the correlations generated by the dynamics of quantum many-particle systems are completely governed by cumulants (9) of groups of operators (7).

Thus, the cumulant structure of solution (8) of the von Neumann hierarchy (2) induces the cumulant structure of solution expansion (27) of the initial-value problem of the quantum nonlinear BBGKY hierarchy for marginal correlation operators.

The evolution equations which satisfy expression (27) are derived similarly to the derivation of hierarchy (21).

We point out that in case of chaos initial data solution expansion (19) of the quantum BBGKY hierarchy (17) for marginal density operators differs from solution expansion (27) of the nonlinear quantum BBGKY hierarchy (21) for marginal correlation operators only by the order of the cumulants of the groups of operators of the von Neumann equations [10; 15]

$$F_s(t,Y) = \sum_{n=0}^{\infty} \frac{1}{n!} \mathbf{Tr}_{s+1,\ldots,s+n} \mathfrak{A}_{1+n}(t,\{Y\},X \setminus Y) \prod_{i=1}^{s+n} F_1(0,i), \tag{28}$$

where $\mathfrak{A}_{1+n}(t)$ is the $(1+n)th$ -order cumulant [20].

The structure of a solution expansion. The direct method of the construction of a solution of the nonlinear quantum BBGKY hierarchy (21) in the form of non-perturbative expansion consists in its derivation on the basis of expansions (16) from non-perturbative solution (19) of initial-value problem of the quantum BBGKY hierarchy (17)—(18). Following stated above approach, we derive a formula for a solution of the quantum nonlinear BBGKY hierarchy for marginal correlation operators in case of general initial data on the basis of definition (14) and non-perturbative solution (8) of initial-value problem of the von Neumann hierarchy (2)—(3). With this aim on $f_n \in \mathfrak{L}^1(\mathcal{H}_n)$ we introduce an analogue of the annihilation operator

$$(\mathfrak{a}f)_s(1,\ldots,s)=\boldsymbol{Tr}_{s+1}f_{s+1}(1,\ldots,s,s+1),\ s\geq 1, \tag{29}$$

and, therefore we have

$$\left(e^{\pm\mathfrak{a}}f\right)_s(1,\ldots,s)=\sum_{n=0}^{\infty}\frac{(\pm 1)^n}{n!}\boldsymbol{Tr}_{s+1,\ldots,s+n}f_{s+n}(1,\ldots,s+n).$$

According to definition (14) of the marginal correlation operators, i.e.

$$G(t)=e^{\mathfrak{a}}g(t),$$

where the sequence $g(t)$ is a solution of the von Neumann hierarchy for correlation operators defined by group (8), i.e.

$$g(t)=\mathcal{G}\left(t|g(0)\right),$$

and to the equality: $g(0)=e^{-\mathfrak{a}}G(0)$, we finally derive

$$G(t)=e^{\mathfrak{a}}\mathcal{G}\left(t|e^{-\mathfrak{a}}G(0)\right). \tag{30}$$

To set down formula (29) in componentwise form we observe, that the following equality holds

$$\prod_{X_i\subset P}\left(e^{-\mathfrak{a}}G(0)\right)_{|X_i|}(X_i)=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\times$$

$$\times\boldsymbol{Tr}_{s+n+1,\ldots,s+n+k}\sum_{k_1=0}^{k}\frac{k!}{k_1!(k-k_1)!}\cdots\sum_{k_{|P|-1}=0}^{k_{|P|-2}}\frac{k_{|P|-2}!}{k_{|P|-1}!\left(k_{|P|-2}-k_{|P|-1}\right)!}\times \tag{31}$$

$$G_{|X_1|+k-k_1}\left(0,X_1,s+n+1,\ldots,s+n+k-k_1\right)\ldots$$

$$G_{\left|X_{|P|}\right|+k_{|P|-1}}\left(0,X_{|P|},s+n+k-k_{|P|-1}+1,\ldots,s+n+k\right).$$

Then according to formulas (29) and (8), for $s\geq 1$, we have

$$G_s(t,Y)=\sum_{n=0}^{\infty}\frac{1}{n!}\boldsymbol{Tr}_{s+1,\ldots,s+n}\sum_{P:\,(1,\ldots,s+n)=\bigcup_i X_i}\times$$

$$\times\mathfrak{A}_{|P|}\left(t,\{X_1\},\ldots,\{X_{|P|}\}\right)\prod_{X_i\subset P}\left(e^{-\mathfrak{a}}G(0)\right)_{|X_i|}(X_i),$$

where $\mathfrak{A}_{|P|}(t)$ is $|P|\,th$-order cumulant (9), and as a result for sequence (29) we obtain

$$G_s(t,1,\ldots,s)=\sum_{n=0}^{\infty}\frac{1}{n!}\boldsymbol{Tr}_{s+1,\ldots,s+n}\sum_{k=0}^{n}(-1)^k\frac{n!}{k!(n-k)!}\times$$
$$\times\sum_{P:\,(1,\ldots,s+n-k)=\bigcup_i X_i}\mathfrak{A}_{|P|}\left(t,\{X_1\},\ldots,\{X_{|P|}\}\right)\times$$
$$\times\sum_{k_1=0}^{k}\frac{k!}{k_1!(k-k_1)!}\ldots\sum_{k_{|P|-1}=0}^{k_{|P|-2}}\frac{k_{|P|-2}!}{k_{|P|-1}!\left(k_{|P|-2}-k_{|P|-1}\right)!}\times \tag{32}$$
$$\times G_{|X_1|+k-k_1}\left(0,X_1,\,s+n-k+1,\ldots,s+n-k_1\right)\ldots$$
$$\ldots G_{\left|X_{|P|}\right|+k_{|P|-1}}\left(0,X_{|P|},s+n-k_{|P|-1}+1,\ldots,s+n\right).$$

Consequently the solution expansion of the nonlinear quantum BBGKY hierarchy has the following structure

$$G_s(t,Y)=\sum_{n=0}^{\infty}\frac{1}{n!}\boldsymbol{Tr}_{s+1,\ldots,s+n}U_{1+n}\left(t;\{Y\},s+1,\ldots,s+n\middle|G(0)\right), \tag{33}$$

where we introduce the notion of the $(n+1)th$-order reduced cumulant $U_{1+n}(t)$ of nonlinear groups of operators (8)

$$U_{1+n}\left(t;\{Y\},s+1,\ldots,s+n\middle|G(0)\right)=$$
$$=\sum_{k=0}^{n}(-1)^k\frac{n!}{k!(n-k)!}\sum_{P:\,(\theta(\{1,\ldots,s\}),s+1,\ldots,s+n-k)=\bigcup_i X_i}\times$$
$$\times\mathfrak{A}_{|P|}\left(t,\{X_1\},\ldots,\{X_{|P|}\}\right)\times$$
$$\times\sum_{k_1=0}^{k}\frac{k!}{k_1!(k-k_1)!}\ldots\sum_{k_{|P|-1}=0}^{k_{|P|-2}}\frac{k_{|P|-2}!}{k_{|P|-1}!\left(k_{|P|-2}-k_{|P|-1}\right)!}\times \tag{34}$$
$$\times G_{|X_1|+k-k_1}\left(0,X_1,\,s+n-k+1,\ldots,s+n-k_1\right)\ldots$$
$$\ldots G_{\left|X_{|P|}\right|+k_{|P|-1}}\left(0,X_{|P|},s+n-k_{|P|-1}+1,\ldots,s+n\right)$$

We give simplest examples of reduced nonlinear cumulants (33):

$$U_1\left(t;\{Y\}|G(0)\right)=\mathcal{G}\left(t;Y|G(0)\right)=$$
$$=\sum_{P:\,(1,\ldots,s)=\bigcup_i X_i}\mathfrak{A}_{|P|}\left(t,\{X_1\},\ldots,\{X_{|P|}\}\right)\prod_{X_i\subset \boldsymbol{P}}G_{|X_i|}\left(0,X_i\right),$$

$$U_2\left(t;\{Y\},s+1\middle|G(0)\right)=$$
$$=\sum_{P:\,(Y,s+1)=\bigcup_i X_i}\mathfrak{A}_{|P|}\left(t,\{X_1\},\dots,\{X_{|P|}\}\right)\prod_{X_i\subset P}G_{|X_i|}\left(0,X_i\right)-$$
$$-\sum_{P:\,(1,\dots,s)=\bigcup_i X_i}\mathfrak{A}_{|P|}\left(t,\{X_1\},\dots,\{X_{|P|}\}\right)\times$$
$$\times\sum_{j=1}^{|P|}\prod_{\substack{X_i\subset P,\\ X_i\neq X_j}}G_{|X_i|}\left(0,X_i\right)G_{|X_j|+1}\left(0,X_j,s+1\right).$$

We remark that in case of solution expansion (19) of the quantum BBGKY hierarchy, an analog of reduced cumulant (33) is the reduced cumulant of groups of operators (7) defined by formula [12]

$$U_{1+n}\left(t;\{Y\},s+1,\dots,s+n\right)=\sum_{k=0}^{n}(-1)^k\frac{n!}{k!(n-k)!}\mathcal{G}_{s+n-k}(-t).$$

Reduced cumulants of nonlinear groups of operators. We indicate some properties of reduced nonlinear cumulants (33) of groups of operators (8). According to formula (32) and properties of cumulants (9), namely $\mathfrak{A}_n(0)=I\delta_{n,1}$, the following equality holds

$$U_{1+n}\left(0;\{Y\},s+1,\dots,s+n\mid G(0)\right)=$$
$$=\sum_{k=0}^{n}(-1)^k\frac{n!}{k!(n-k)!}\mathfrak{A}_1\left(0,\{1,\dots,s+n-k\}\right)G_{s+n}\left(0,1,\dots,s+n\right)= \quad (35)$$
$$=G_{s+n}\left(0,1,\dots,s+n\right)\delta_{n,0},$$

and hence the marginal correlation operators determined by series (32) satisfy initial data (22).

In case of $n=0$ for $f\in\mathfrak{L}_0^1(\mathcal{F}_{\mathcal{H}})$ in the sense of the norm convergence of the space $\mathfrak{L}^1(\mathcal{H}_s)$ the infinitesimal generator of first-order reduced cumulant (33) coincides with generator (23) of the von Neumann hierarchy (2)

$$\lim_{t\to 0}\frac{1}{t}\Big(\big(U_1\left(t;\{Y\}\mid f\right)\big)-f_s(Y)\Big)=\mathcal{N}\left(Y\mid f\right),\ s\geq 1,$$

where the operator $\mathcal{N}(Y\mid f)$ is defined by formula (23). In case of n=1 for second-order reduced cumulant (33) in the same sense we obtain the following equality

$$\boldsymbol{Tr}_{s+1}\lim_{t\to 0}\frac{1}{t}U_2\left(t;\{Y\},s+1\middle|f\right)=$$

$$= \sum_{i \in Y} \boldsymbol{Tr}_{s+1} \left(-\mathcal{N}_{\mathrm{int}}\left(i, s+1\right)\right)\Big(f_{s+1}\left(t, Y, s+1\right) +$$

$$+ \sum_{\substack{P:(Y,s+1)=X_1 \bigcup X_2, \\ i \in X_1; s+1 \in X_2}} f_{|X_1|}\left(t, X_1\right) f_{|X_2|}\left(t, X_2\right)\Big),$$

where notations are used as above for hierarchy (21), and for $n \geq 2$ as a consequence of the fact that we consider a system of particles interacting by a two-body potential, it holds

$$\boldsymbol{Tr}_{s+1,\ldots,s+n} \lim_{t \to 0} \frac{1}{t} U_{1+n}\left(t; \{Y\}, s+1, \ldots, s+n \mid f\right) = 0.$$

In case of initial data satisfying a chaos property, i.e. $G(0) \equiv \left(0, G_1(0,1), 0, \ldots\right)$, for the $\left(1+n\right)th$ -order reduced cumulant we have

$$U_{1+n}\left(t; \{Y\}, s+1, \ldots, s+n \mid G^{(1)}\left(0\right)\right) = \mathfrak{A}_{s+n}\left(t, 1, \ldots, s+n\right) \prod_{i=1}^{s+n} G_1\left(0, i\right),$$

i.e. the only summand that gives contribution to the result is the one with k=0 and $|P| = s+n$, since otherwise there is at least one operator $G_s\left(0\right)$ with $s \geq 2$ in the last product.

For the $\left(1+n\right)th$ -order reduced cumulant (33) the following inequality holds

$$\left\| U_{1+n}\left(t; \{Y\}, s+1, \ldots, s+n \mid f\right) \right\|_{\mathfrak{L}^1\left(\mathcal{H}_{s+n}\right)} \leq 2n!s!\left(2e^3\right)^{s+n} c^{s+n}, \quad (36)$$

where

$$\boldsymbol{c} \equiv \max_{P:\left(1,\ldots,s+n-k\right)=\bigcup_i X_i} \; \max_{k, k_1, \ldots, k_{|P|-1} \in \left(s+n-k+1, \ldots, s+n\right)}$$

$$\left(\left\| f_{|X_1|+k-k_1} \right\|_{\mathfrak{L}^1\left(\mathcal{H}_{|X_1|+k-k_1}\right)}, \ldots, \left\| f_{|X_{|P|}|+k_{|P|-1}} \right\|_{\mathfrak{L}^1\left(\mathcal{H}_{|X_{|P|}|+k_{|P|-1}}\right)} \right).$$

To prove this inequality we first remark that for cumulant (9) the following estimate holds

$$\left\| \mathfrak{A}_{|P|}\left(t, \{X_1\}, \ldots, \{X_{|P|}\}\right) f_n \right\|_{\mathfrak{L}^1\left(\mathcal{H}_n\right)} \leq |P|!\, e^{|P|} \parallel f_n \parallel_{\mathfrak{L}^1\left(\mathcal{H}_n\right)}. \quad (37)$$

Indeed, we have

$$\left\| \mathfrak{A}_{|P|}\left(t, \{X_1\}, \ldots, \{X_{|P|}\}\right) f_n \right\|_{\mathfrak{L}^1\left(\mathcal{H}_n\right)} \leq \sum_{P^{'}:\left(\{X_1\}, \ldots, \{X_{|P|}\}\right)=\bigcup_k Z_k} \left(|P^{'}|-1\right)! \times$$

$$\times\left\| \prod_{Z_k\subset P'} \mathcal{G}_{|\theta(Z_k)|}\left(-t,\theta(Z_k)\right) f_n \right\|_{\mathfrak{L}^1(\mathcal{H}_n)} = \| f_n \|_{\mathfrak{L}^1(\mathcal{H}_n)} \sum_{l=1}^{|P|} s\left(|P|,l\right)\left(l-1\right)!,$$

where $\boldsymbol{s}\left(|P|,l\right)$ are the Stirling numbers of second kind and we use the isometric property of the groups $\mathcal{G}_n\left(-t\right), n\geq 1$. Estimate (36) holds as a consequence of the inequality

$$\sum_{l=1}^{|P|} \boldsymbol{s}\left(|P|,l\right)\left(l-1\right)! \leq |P|!\, e^{|P|}.$$

Then owing to estimate (36), for the $(1+n)th$-order reduced cumulant (33) we have

$$\left\| U_{1+n}\left(t;\{Y\},s+1,\ldots,s+n \mid f\right)\right\|_{\mathfrak{L}^1(\mathcal{H}_{s+n})} \leq$$

$$\leq \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} \sum_{P:\,(1,\ldots,s+n-k)=\bigcup_i X_i} |P|!\, e^{|P|} \sum_{k_1=0}^{k} \frac{k!}{k_1!\left(k-k_1\right)!}\cdots$$

$$\ldots \sum_{k_{|P|-1}=0}^{k_{|P|-2}} \frac{k_{|P|-2}!}{k_{|P|-1}!\left(k_{|P|-2}-k_{|P|-1}\right)!} \left\| f_{|X_1|+k-k_1} \right\|_{\mathfrak{L}^1\left(\mathcal{H}_{|X_1|+k-k_1}\right)} \cdots \left\| f_{|X_{|P|}|+k_{|P|-1}} \right\|_{\mathfrak{L}^1\left(\mathcal{H}_{|X_{|P|}|+k_{|P|-1}}\right)} \leq$$

$$\leq \sum_{k=0}^{n} \frac{n!}{\left(n-k\right)!} \sum_{P:\,(1,\ldots,s+n-k)=\bigcup_i X_i} |P|!\, e^{2|P|-1} \boldsymbol{c}^{|P|}.$$

As result of using of the definition of the Stirling numbers of second kind $\boldsymbol{s}\left(s+n-k,l\right)$ and the inequalities

$$\sum_{k=0}^{n} \frac{n!}{\left(n-k\right)!} \sum_{P:\,(1,\ldots,s+n-k)=\bigcup_i X_i} |P|!\, e^{2|P|-1} =$$

$$= \sum_{k=0}^{n} \frac{n!}{\left(n-k\right)!} \sum_{l=1}^{s+n-k} \boldsymbol{s}\left(s+n-k,l\right) l!\, e^{2l-1} \leq$$

$$\leq \sum_{k=0}^{n} \frac{n!\left(s+n-k\right)!}{\left(n-k\right)!} e^{3(s+n-k)} \leq 2n!\,s!\left(2e^3\right)^{s+n},$$

we obtain estimate (36).

Thus, according to estimate (36), for initial data from the space $\mathfrak{L}^1\left(\mathcal{H}_n\right)$ series (32) converges provided that $\max_{n\geq 1} \| G_n\left(0\right) \|_{\mathfrak{L}^1(\mathcal{H}_n)} < \left(2e^3\right)^{-1}$, and the following inequality holds

$$\| G_s(t) \|_{\mathfrak{L}^1(\mathcal{H}_s)} < 2s!\left(2e^3\right)^s . \tag{38}$$

A solution of the Cauchy problem of the nonlinear quantum BBGKY hierarchy for marginal correlation operators (21) is determined by the following one-parametric mapping

$$\mathbb{R} \ni t \to \mathcal{U}(t \mid f) = e^{\mathfrak{a}}\mathcal{G}\left(t \mid e^{-\mathfrak{a}} f\right), \tag{39}$$

which is defined on the space $\mathfrak{L}^1(\mathcal{F}_{\mathcal{H}})$ owing estimate (38), and has the group property

$$\mathcal{U}\left(t_1 \mid \mathcal{U}(t_2 \mid f)\right) = \mathcal{U}\left(t_2 \mid \mathcal{U}(t_1 \mid f)\right) = \mathcal{U}(t_1 + t_2 \mid f).$$

Indeed, according to definition (29) and taking to attention the group property of the mapping $\mathcal{G}(t \mid \cdot)$, we obtain

$$\mathcal{U}(t_1 + t_2 \mid f) = e^{\mathfrak{a}}\mathcal{G}\left(t_1 + t_2 \mid e^{-\mathfrak{a}} f\right) = e^{\mathfrak{a}}\mathcal{G}\left(t_1 \mid \mathcal{G}\left(t_2 \mid e^{-\mathfrak{a}} f\right)\right) =$$

$$= e^{\mathfrak{a}}\mathcal{G}\left(t_1 \mid e^{-\mathfrak{a}} e^{\mathfrak{a}}\mathcal{G}\left(t_2 \mid e^{-\mathfrak{a}} f\right)\right) = e^{\mathfrak{a}}\mathcal{G}\left(t_1 \mid e^{-\mathfrak{a}}\mathcal{U}(t_2 \mid f)\right) = \mathcal{U}\left(t_1 \mid \mathcal{U}(t_2 \mid f)\right).$$

To construct the generator of the strong continuous group $\mathcal{U}(t, Y \mid \cdot)$ we differentiate it in the sense of the norm convergence on the space $\mathfrak{L}^1(\mathcal{H}_s)$

$$\frac{d}{dt}\mathcal{U}(t; Y \mid f)\Big|_{t=0} = \frac{d}{dt}\left(e^{\mathfrak{a}}\mathcal{G}\left(t \mid e^{-\mathfrak{a}} f\right)\right)_s (Y)\Big|_{t=0} =$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} \boldsymbol{Tr}_{s+1,\ldots,s+n} \, \mathcal{N}\left(X \mid \mathcal{G}\left(t \mid e^{-\mathfrak{a}} f\right)\right)\Big|_{t=0} = \left(e^{\mathfrak{a}}\mathcal{N}\left(\cdot \mid e^{-\mathfrak{a}} f\right)\right)_s (Y)$$

where $\mathcal{N}(\cdot \mid f)$ is a generator of the von Neumann hierarchy (2) defined by formula (4) on the subspaces $\mathfrak{L}^1_0(\mathcal{H}_s) \subset \mathfrak{L}^1(\mathcal{H}_s)$, $s \geq 1$, or in the componentwise form in case of a two-body interaction potential

$$\begin{aligned} &\left(e^{\mathfrak{a}}\mathcal{N}\left(\cdot \middle| e^{-\mathfrak{a}} f\right)\right)_s (Y) = \mathcal{N}\left(Y \middle| f\right) + \\ &+ \boldsymbol{Tr}_{s+1} \sum_{i \in Y} \left(-\mathcal{N}_{\mathrm{int}}(i, s+1)\right)\Big(f_{s+1}(Y, s+1) + \\ &+ \sum_{\substack{P:(Y, s+1) = X_1 \bigcup X_2, \\ i \in X_1; s+1 \in X_2}} f_{|X_1|}(X_1) f_{|X_2|}(X_2)\Big), \end{aligned} \tag{40}$$

where we use notations as above for formula (21). Formula (40) describes the structure of the infinitesimal generator of mapping (39) in the general case of many-body interaction potentials.

Thus, for abstract initial-value problem for hierarchy (22) in the space $\mathfrak{L}^1(\mathcal{F}_{\mathcal{H}})$ the following theorem is true.

If $\max_{n\geq 1} \| G_n(0) \|_{\mathfrak{L}^1\left(\mathcal{H}_n\right)} < \left(2e^3\right)^{-1}$, then for $t \in \mathbb{R}$ a solution of the initial-value problem (22)—(23) of the nonlinear quantum BBGKY hierarchy is determined by expansion (32). If $G_n(0) \in \mathfrak{L}_0^1\left(\mathcal{H}_n\right) \subset \mathfrak{L}^1\left(\mathcal{H}_n\right)$ it is a strong (classical) solution and for arbitrary initial data $G_n(0) \in \mathfrak{L}^1\left(\mathcal{H}_n\right)$ it is a weak (generalized) solution.

The proof of the theorem is similarly to the prove of an existence theorem for the von Neumann hierarchy [10].

Conclusion. In the paper the origin of the microscopic description of non-equilibrium correlations of quantum many-particle systems obeying the Maxwell-Boltzmann statistics has been considered. The nonlinear quantum BBGKY hierarchy (22) for marginal correlation operators was introduced. It gives an alternative approach to the description of the state evolution of quantum infinite-particle systems in comparison with quantum BBGKY hierarchy for marginal density operators. The evolution of both finitely and infinitely many quantum particles is described by initial-value problem of the nonlinear quantum BBGKY hierarchy (21) and for finitely many particles the nonlinear quantum BBGKY hierarchy is equivalent to the von Neumann hierarchy(2).

A non-perturbative solution of the nonlinear quantum BBGKY hierarchy is constructed in the form of expansion (32) over particle clusters which evolution is governed by corresponding-order cumulant (33) of the nonlinear groups of operators generated by solution (8) of the von Neumann hierarchy (2). We established that in case of absence of correlations at initial time the correlations generated by the dynamics of quantum many-particle systems (27) are completely determined by cumulants (9) of groups of operators (7).

Thus, the cumulant structure of solution (8) of the von Neumann hierarchy (2) induces the cumulant structure of solution expansion (32) of initial-value problem of the nonlinear quantum BBGKY hierarchy (22).

We emphasize that intensional Banach spaces for the description of states of infinite-particle systems, which are suitable for the description of the kinetic evolution or equilibrium states, are different from the exploit spaces [12; 14]. Therefore marginal correlation operators from the space $\mathfrak{L}^1\left(\mathcal{F}_{\mathcal{H}}\right)$ describe finitely many quantum particles. In order to describe the evolution of infinitely many particles we have to construct solutions for initial data from more general Banach spaces than the space of sequences of trace class operators. For example, it can be the space of sequences of bounded translation invariant operators which contains the marginal density operators of equilibrium states. In that case every term of the solution expansion of the nonlinear quantum BBGKY hierarchy (22) contains the divergent traces, which can be renormalized due to the cumulant structure of solution expansion (33).

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Робота присвячена проблемі опису нерівноважних кореляцій квантових багаточастинкових систем. Побудовано розв'язок задачі Коші нелінійної квантової ієрархії рівнянь ББГКІ у формі розкладу по групах частинок, еволюція яких описується відповідного порядку кумулянтом груп нелінійних операторів ієрархії рівнянь фон Неймана.

Ключові слова: *нелінійна квантова ієрархія ББГКІ, ієрархія фон Неймана, кореляційний оператор, квантові багаточастинкові системи.*

Отримано: 06.05.2011

УДК 517.5

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ТЕОРЕМИ ІСНУВАННЯ ЕКСТРЕМАЛЬНОГО ЕЛЕМЕНТА ДЛЯ ЗАДАЧІ НАЙКРАЩОЇ У РОЗУМІННІ ОПУКЛОЇ НЕПЕРЕРВНОЇ ФУНКЦІЇ РІВНОМІРНОЇ АПРОКСИМАЦІЇ НЕПЕРЕРВНОГО КОМПАКНОЗНАЧНОГО ВІДОБРАЖЕННЯ

Доведено деякі теореми існування екстремального елемента для задачі найкращої у розумінні опуклої неперервної функції рівномірної апроксимації неперервного компакнозначного відображення множинами неперервних однозначних відображень.

Ключові слова: *найкраща у розумінні опуклої неперервної функції рівномірна апроксимація, компакнозначне відображення, екстремальний елемент, теореми існування.*

Вступ. У статті для задачі найкращої у розумінні опуклої неперервної функції рівномірної апроксимації неперервного компактнозначного відображення множинами неперервних однозначних відображень доведено деякі теореми існування екстремального елемента, які узагальнюють на випадок цієї задачі відповідні теореми існування екстремального елемента для задачі найкращого у розумінні опуклої функції наближення елемента лінійного нормованого простору опуклою множиною цього простору, встановлені у праці [1], розглянуто допоміжні твердження, які представляють і самостійний інтерес.

Постановка задачі. Нехай S -компакт, X -лінійний над полем дійсних чисел нормований простір, $C(S,X)$ — лінійний над полем дійсних чисел нормований простір однозначних відображень g компакта S в X, неперервних на S, з нормою $\|g\| = \max_{s \in S} \|g(s)\|, K(X)$ — су-