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ALGORITHM FOR SOLVING SYSTEMS WITH BAND MATRICES IN THE PROBLEMS OF PREDICTING THE SERVICE LIFE OF WELDED STRUCTURES

New efficient algorithmic and software tools for solving systems of linear algebraic equations of super large orders with an assessment of the reliability of the results obtained have been developed. The application of the developed algorithms is aimed at numerical prediction of the bearing capacity, residual life and serviceability of critical welded structures, in particular, main pipelines and pressure vessels with identified operational defects. The use of specialized computational methods not only improves the accuracy of residual life estimation, but also optimizes the processes of technical diagnostics and life extension of critical engineering facilities.

Key words: *systems of linear algebraic equations, parallel algorithms, supercomputers, numerical prediction.*

Introduction. The emergence of dispersed metal deformation may be attributed to a number of factors of various natures, such as: intense plastic deformation, fatigue loading, high concentration of diffusible hydrogen, radiation exposure, and so on. In this regard, the accumulation of subcritical microdefect (SMD) under regular operational influences is considered as material degradation in the development of design solutions, allowing for a conservative assessment of the current state of structures. However, if the structure has been subjected to significant irregular influences of a natural or anthropogenic nature (landslides, earthquakes, overloads during commissioning and adjustment work, etc.), it is necessary to assess the damage degree that the material has sustained. The presence of installation or repair welds complicates such analysis because welded joints are areas of local chemical and structural inhomogeneity and are characterized by a residual stress-strain state (SSS) caused by accumulated plastic deformations during welding, which depend on the technological parameters of the welding process.

Therefore, the development of mathematical models for the accumulation of SMD in critical welded structures, particularly for typical cases of

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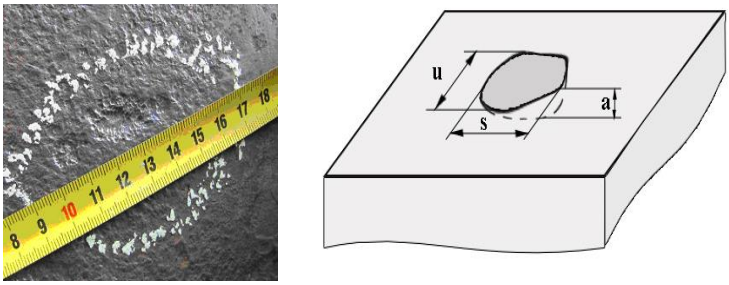
significant external static, statically variable, or fatigue influences that cause plastic flow of the metal and disruption of its integrity, is both relevant and practically important.

In researches [7, 8], approaches to numerical analysis of the limit state of welded pipeline elements with defects from local metal corrosion in the weld metal and heat-affected zone (HAZ) were proposed. These approaches investigated the effect of post-weld stress state on the tendency of the structure to initiate and develop failure in the defect area. In particular, it was shown that the interaction between stress and plastic deformation fields in the region of geometric concentrators and weld locations can negatively impact the static strength of the structure. However, important practical tasks related to assessing the operability of pipeline elements with corrosion defects in the area of installation or repair welding under ultra-low cycle and low cycle loading conditions require further development of these methodologies.

The development of mathematical models for the processes of initiation and growth of damage mechanisms in the metal of critical welded structures under the influence of ultra-low cycle and low cycle loading is relevant for assessing their condition. Additionally, it is important to research, using the example of pipeline elements with corrosion defects in the area of the mounting or repair weld, the specific characteristics of the impact of welding on the performance of defective structures.

Main part. *Defects of local metal loss of pipelines, methods of their schematization and criteria for admissibility.* Defects of corrosion metal loss can be divided into local and general thinning. Local corrosion metal loss is characterized by comparable sizes (length, width, depth) of defects, and general thinning has a much larger length and/or width compared to the depth. Local corrosion metal loss is schematized by a semi-elliptical geometric anomaly (Fig. 1), general corrosion is considered as a part of the structure with a smaller wall thickness.



(a) *(b)*

Fig. 1. *Appearance (a) and schematic representation (b) of a typical defect of local corrosion loss of metal*

The acceptability of defects in the local thinning of the pipeline wall can be based on experimental researches of the limit state of defective structures, which is implemented in most relevant regulatory documents, or on the research of the acceptability of the stress-strain state of structural elements, taking into account their actual damage. The absence of sharp concentrators in the zone of local metal loss makes it possible to determine the stress distribution with high accuracy. Based on the calculated stress and strain fields, a boundary analysis of the defective welded structure is performed. There are three main options for such an analysis:

- assessment of local stress and strain tolerance;
- assessment of the permissibility of the ultimate load for the actual state of the structure;
- determination of the probability of failure.

It should be noted that the first approach is the most conservative, since it considers the limit state of the structure as a characteristic of the most loaded point (volume). From this point of view, the integral criteria of the second and third assessment options are more adequate to the specific state of pipeline damage and allow for less conservative assessments.

Within the second approach, the main mechanism of failure of the welded structure is the viscous mechanism, which determines the ultimate operating load that the pipeline can withstand in the actual state of corrosion damage. In this case, the conclusion about the acceptability of the structural condition is based on the known partial safety factors for each component of the operating load within the framework of the limit assessment of the complex force impact.

The mathematical consideration of the combined problem of temperature field kinetics, stress and strain development, and micropore formation is based on a finite element description using eight-nodefinite elements (FE, Fig. 2). Within the volume limited by the element under consideration, the distribution of temperatures, stresses and strains is assumed to be homogeneous.

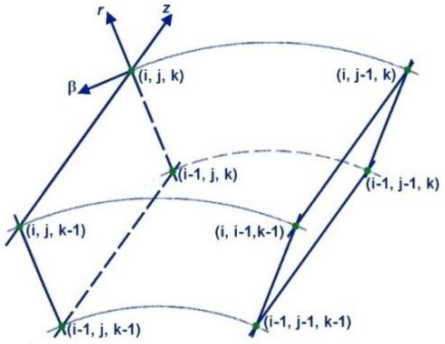


Fig. 2. *Diagram of a finite element in the coordinate system* r, β, z

The finite-element solution of the boundary value problem of unsteady thermoplasticity was carried out by studying the joint development of elastic-plastic deformation, subcritical and critical fracture by a viscous mechanism. The implementation of such a numerical study is associated with two nonlinearities in physical processes: plastic deformation and fracture. For the formal solution of these nonlinear problems, the corresponding iterative processes are implemented to find the state of the FE that satisfies the equilibrium equations and the condition:

$$\Phi = \left(\sigma_i / \sigma_T\right)^2 - \left(q_3 f^*\right)^2 + 2q_1 f^* \cosh\left(q_2 \frac{3\sigma_m}{2\sigma_T}\right) - 1, \qquad (1)$$

where $q_1 = 1,5$, $q_2 = 1$, $q_3 = 1,5$ – constants, f^* – equivalent pore concentration, σ_T – fluidity border of the material, $\sigma_m = \frac{\sigma_{rr} + \sigma_{\beta\beta} + \sigma_{zz}}{3}$ – average value of normal components of the stress tensor, $\sigma_I = (0,5\sigma_{ij}\sigma_{ij})^{0,5}$ – stress intensity. The equivalent pore concentration f^* in (1), which takes into account the interaction between individual discontinuities, is estimated based on the following relations:

$$f^* = \begin{cases} f, & \text{if} \quad f \le f_c, \\ f_c + \dfrac{f_u^* - f_c}{f_F - f_c}(f - f_c), & \text{if} \quad f > f_c, \end{cases}$$

where f_c – critical concentration of discontinuities, up to which individual pores do not interact, is generally considered to be $f_c = 0,15$; f_F – the pore concentration at which the finite element fractures, $f_u^* = \frac{1}{q_1}$.

Thus, to solve the nonlinearity in plastic deformation, the approach proposed by Makhnenko V. I. [4] was used, namely, consideration of the material state function Ψ, which satisfies the following conditions on the yield surface:

$$\begin{aligned} \Psi &= \frac{1}{2G}, \quad if \quad \sigma_i < \sigma_s, \\ \Psi &> \frac{1}{2G}, \quad if \quad \sigma_i = \sigma_s, \end{aligned} \qquad (2)$$

where $\sigma_s = \sigma_T \sqrt{1 + (q_3 f^*)^2 - 2q_1 f^* \cosh(1,5 q_2\, \sigma_m / \sigma_T)}$, $G = \frac{0,5E}{1+\nu}$, E is Young's modulus, ν is Poisson's ratio; and the condition $\sigma_i > \sigma_s$ is inadmissible.

The main difficulty in modeling cyclic loading is that small changes in the metal during one loading cycle, namely the accumulation and growth of DM, cause a change in the yield surface, which causes a change in the plastic deformation loop. However, at each stage of loading it is necessary to determine the equilibrium state of damage and the corresponding distribution of stresses and strains. For this, assuming that the steady state is characterized by neglecting the low growth rate of DM, it is proposed to carry out such an iterative process according to the function $\Psi_K = f_0 K_1 \exp(K_2 \sigma_m/\sigma_i) d\varepsilon_i^p$:

$$F = \begin{cases} F + dF, & \text{if} \quad \Psi_K \le \Psi_K^0 \approx 10^{-5}, \\ F, & \text{if} \quad \Psi_K > \Psi_K^0, \end{cases} \tag{3}$$

where F is the system of external force loads acting on the structure; dF is the increase in force loads during the numerical study.

Thus, solving problems (2), (3) at each stage of the study allows us to determine the damage degree to the material of the welded structure with a controlled degree of accuracy, taking into account its geometric features. As a criterion for the initiation of macroscopic fracture, we use the conditions of brittle-viscous fracture, namely, the fulfilment of one of three conditions:

$$\left(\Psi - \frac{1}{2G}\right)_{KP} \ge \frac{\varepsilon_f - \varepsilon_p^*}{1,5\sigma_i} \approx \frac{\varepsilon_f - \varepsilon_p^*}{1,5\sigma_s(\varepsilon_p, T)},$$

or

$$f^* \to f_d^* = 2(q_1/q_3)\cosh(1,5 q_2 \sigma_m/\sigma_T), \tag{4}$$

or

$$\frac{3\sigma_1}{3-2f} > S_K,$$

where S_K is the value of the microscope stress, ε_f is the ultimate deformation capacity of the material, the index «*» refers to the variable in the previous step of the research.

If one of the conditions (4) is met for a certain structural element, it is considered that this structural element loses its bearing capacity and a macroscopic discontinuity has formed in its place. Further loading of the structure and the development of macro-destruction, as a result, leads to an avalanche-like loss of the bearing capacity of the material within the iterative process (3), which can be interpreted as a spontaneous destruction of the structure. Depending on the accuracy of the available data and on the production necessity, the limit state of the responsible structure can be considered either the moment of the emergence of the first macro-discontinuity, or its (the structure's) spontaneous destruction.

Plastic deformation is determined by the ratio

$$\Delta\varepsilon_{ij} = \left(\Psi - \frac{1}{2G}\right)(\sigma_{ij} - \delta_{ij}\sigma_m).$$

At each step of the iteration along Ψ, the stresses σ_{ij} are calculated as follows (repeated indices are summed up):

$$\sigma_{ij} = \frac{1}{\Psi}\left(\Delta\varepsilon_{ij} + \delta_{ij}\frac{\Psi - K}{K}\Delta\varepsilon\right) + J_{ij},$$

where

$$\Delta\varepsilon = \frac{\Delta\varepsilon_{ij}}{3},$$

$$J_{ij} = \frac{1}{\Psi}\left(b_{ij} - \delta_{ij}b + \delta_{ij}\left(K\sigma^* - \frac{\Delta\varepsilon_T + \Delta f/3}{K}\right)\right),$$

$$b = \frac{b_{ii}}{3}.$$

The relation between the components of the strain tensor $\Delta\varepsilon_{ij}$ and the displacement increment vector ΔU_i is given by the following expression (comma denotes differentiation within the FE):

$$\Delta\varepsilon_{ij} = \frac{\Delta U_{i,j} + \Delta U_{j,i}}{2}.$$

At each step of the study and iteration along Ψ, it is necessary to solve the system of equations in the variables of the displacement increment vector ΔU_i in the FE nodes, which is determined from the conditions of the minimum of the following functional (using the Lagrange's variational principle):

$$E_I = -\frac{1}{2}\sum_V(\sigma_{ij} + J_{ij})\Delta\varepsilon_{ij}V_{m,n,r} + \sum_{S_P} P_i\Delta U_i\Delta S_P^{m,n,r},$$

where $\sum_V$ is the sum operator over the internal SE, $\sum_{S_P}$ is the sum operator over the surface SE, on which the components of the force vector P_i are given. In other words, a system of equations

$$\frac{\partial E_I}{\partial\Delta U_{m,n,r}} = 0, \quad \frac{\partial E_I}{\partial\Delta V_{m,n,r}} = 0, \quad \frac{\partial E_I}{\partial\Delta W_{m,n,r}} = 0, \tag{5}$$

allows us to obtain a solution in the components of the displacement increment vector at each step of the study and iterations along Ψ for a specific SE.

Block cyclic algorithm for LU-decomposition of a band matrix. The analysis of such technological schemes and the study of computer pro-

grams that implement them show that the lion's share of time is spent on solving system of linear algebraic equations (SLAE) (5) with asymmetric band matrices. As mentioned above, the number of such SLAE reaches several hundred or even thousands. Therefore, it is advisable to use computers with parallel computing, including hybrid architecture, and appropriate algorithmic and software tools to solve these SLAE [1-3, 5].

So, let us consider the algorithm of LU-decomposition of the band matrix SLAE (5) A of order n with m_l under the diagonals and m_u above the diagonals on computers with multi-core processors.

In general case, as a result of the permutations, the number of supradiagonals in the upper triangular matrix U may increase to $m_u + m_l$, the lower triangular matrix L and the vector Pb are not explicitly formed, and the permutations are taken into account when solving the SLAP $Ly = Pb$. The matrix A is divided into square blocks of order s. For the simplification, we will consider that $n = Ns$, $m_l = M_L s$, $m_l + m_u = M_U s$:

$$A = \begin{pmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,M_U+1} & 0 & 0 & \cdots & 0 \\ A_{2,1} & A_{2,2} & \cdots & A_{2,M_U+1} & A_{2,M_U+2} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & & & & & \\ A_{M_L+1,1} & A_{M_L+1,2} & & & & & & \\ 0 & A_{M_L+2,2} & & & & & & \\ 0 & 0 & & & & \ddots & \vdots & \vdots \\ \vdots & \vdots & & & & \cdots & A_{N-1,N-1} & A_{N-1,N} \\ 0 & 0 & & & & \cdots & A_{N,N-1} & A_{N,N} \end{pmatrix}.$$

Let's move on to the description of the algorithm. For each $I = 1, ..., N$, the following steps are performed:

1. *LU*-decomposition with partial selection of the main element of the block column – a rectangular submatrix consisting of blocks $A_{I,I}, A_{I+1,I}, A_{I+2,I}, \ldots, A_{K,I}$, where $K = min(I + M_L, N)$:

$$\begin{pmatrix} A_{I,I} \\ \vdots \\ A_{K,I} \end{pmatrix} = P_I \begin{pmatrix} L_{I,I} \\ \vdots \\ L_{K,I} \end{pmatrix} U_{I,I}. \qquad (6)$$

2. Transpose the rows in those subarrays where necessary, using the matrix P_I.
3. Calculating the row of blocks $U_{I,I+1}, U_{I,I+2}, \ldots, U_{I,J}$ of the matrix U, where $J = min(I + M_U, N)$, as a solution to the matrix SLAE with a lower triangular matrix:

$$L_{I,I}\left(U_{I,I+1} \quad \cdots \quad U_{I,J}\right) = \left(A_{I,I+1} \quad \cdots \quad A_{I,J}\right). \tag{7}$$

4. For $I < Ns$, the rank modification of the blocks $A_{M,L}$, where $M = I+1,\ldots,K$ and $L = I+1,\ldots,J$, according to the formula:

$$A_{M,L} \leftarrow A_{M,L} - L_{M,I}\,U_{I,L}. \tag{8}$$

Note that at step I of this algorithm, operations are performed only with blocks of the submatrix of size $(J+1)s \times (K+1)s$, whose left upper block is $A_{I,I}$.

If the partial selection of the main element is carried out only in the leading diagonal block $A_{I,I}$, then the first point can be divided into two:

a) LU-decomposition $A_{I,I} = P_I L_{I,I} U_{I,I}$;

b) Calculation of the column of blocks $L_{I+1,I}, L_{I+2,I}, \ldots, L_{K,I}$ matrix L as a solution of the matrix SLAE with the lower triangular matrix:

$$U_{I,I}^T \left(L_{I+1,I}^T \quad \cdots \quad L_{K,I}^T\right) = \left(A_{I+1,I}^T \quad \cdots \quad A_{K,I}^T\right).$$

This technique allows to reduce the possible number of supra-diagonals in the upper triangular matrix U to no more than $m_u + s$, which reduces the total number of arithmetic operations. The analysis of formulas (6)-(8) showed that software modules for matrix-matrix operations from hardware developers can be used to implement most of the calculations.

For the parallel algorithm, the rows of blocks of the initial matrix A and the development matrices L, U are distributed cyclically among the processes so that each process has at least one row of blocks modified according to (8) at this step.

The order and efficiency of this algorithm largely depend on the set of elements of the matrix column on which the main element is selected. The most efficient is the option when the master element is selected only among the column elements of the leading diagonal block, less efficient is the selection among the column elements distributed to the leading process, and the least efficient (in terms of the number of operations, exchanges and synchronizations) is the selection among all non-zero column elements.

Testing of the parallel algorithm. The proposed parallel algorithm was tested on solving several SLAEs with tape asymmetric matrices, including those arising from mathematical modeling of various states of welded structures. The effects of computer architecture and algorithm parameters (number of threads, block size, etc.), as well as SLAE parameters (structure, order, band width, etc.) on the solution time were studied. For testing, we used several SLAEs with band matrices, the parameters of which are presented in Table 1.

Table 1

Test matrix set

№	Name	Order n	The number of subdiagonals m_l	The number of supradiagonals m_u
1	A-126-20	126 000	2 001	2 001
2	A-126-09	126 000	902	902
3	A-055-10	55 650	1 052	1 052
4	A-052-10	52 500	1 052	1 052
5	A-137-44	137 826	4 448	4 448
6	A-117-02	117 092	239	239
7	A-798-05	798 624	565	565

Table. 2 shows the times for solving several SLAEs, the parameters of which are given in Table. 2, on computers of different architectures using different algorithms. Here, a personal computer (denoted by PC) was mainly used as a computer with a 1 Intel Core I7 4 core processor, and an 8-node SKIT-5[6] cluster with 192-core nodes on AMD EPYC 7642 with a peak performance of 100 TFlops was used as a parallel computer.

Table 2

Solving times for one system

Matrix SLAE	Solving time (seconds)	
	serial algorithm (1 thread)	parallel algorithm (32 threads)
	PC	SKIT-5
A-126-20	≈2 240	52,32
A-126-09	≈ 660	10,13
A-055-10	≈ 170	4,08
A-052-10	≈ 210	12,28
A-137-44	≈ 7 080	185,70
A-117-02	≈ 60	2,70
A-798-05	≈ 800	35,00

These results, as well as the results of other experiments, have shown that the use of parallel computing can significantly reduce the time for solving problems by 15 to 60 times.

Experimental research of parallel algorithms. Fig. 3 shows the acceleration for different numbers of flows obtained by solving two SLAEs with asymmetric band matrices (A-126-09 and A-055-10) that arise in problems of determining the critical load of a thick-walled pipeline element with a wall thinning defect. The SLAE was solved on one node of the SCIT cluster using the developed software modules. The accelerations were calculated in relation to the solution time on a personal computer using a sequential algorithm of the Gauss method.

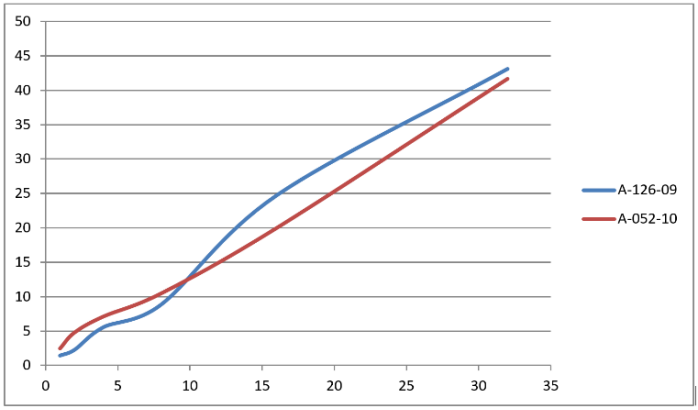


Fig. 3. *Acceleration when using developed software modules*

Conclusions. In this paper, we develop a parallel algorithm for solving systems of linear algebraic equations of super large orders with sparse data structure. The algorithm is optimized for modern parallel and hybrid computing systems, which allows to significantly increasing the solution speed due to efficient parallelization of calculations and rational use of memory.

Testing on real engineering problems, in particular, to assess the remaining service life of pipelines and pressure vessels with operational defects, confirmed the high accuracy of calculations and the performance of the proposed approach.

The results obtained indicate the prospects of using the developed methods for high-precision modeling of complex physical and mechanical processes in real time, which is important for ensuring the reliability and safety of critical engineering facilities.

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АЛГОРИТМ РОЗВ'ЯЗУВАННЯ СИСТЕМ ЗІ СТРІЧКОВИМИ МАТРИЦЯМИ В ЗАДАЧАХ ПРОГНОЗУВАННЯ РЕСУРСУ ЗВАРНИХ КОНСТРУКЦІЙ

Розроблено нові ефективні алгоритмічні та програмні засоби розв'язування систем лінійних алгебраїчних рівнянь надвеликих порядків з оцінкою достовірності отриманих результатів. Застосування розроблених алгоритмів спрямоване на чисельне прогнозування несучої здатності, залишкового ресурсу та працездатності відповідальних зварних конструкцій, зокрема, магістральних трубопроводів та посудин, що працюють під тиском, з виявленими експлуатаційними дефектами. Використання спеціалізованих обчислювальних методів не тільки підвищує точність оцінки залишкового ресурсу, але й оптимізує процеси технічної діагностики та подовження ресурсу відповідальних інженерних об'єктів.

Ключові слова: *системи лінійних алгебраїчних рівнянь, паралельні алгоритми, гібридні обчислення, суперкомп'ютери, чисельне прогнозування.*

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