The temperature regimes of a copper panel during its induction heating by a homogeneous quasi-steady electromagnetic field are analyzed numerically. Two characteristic modes of near-surface and continuous induction heating of the panel are considered. The results of calculations of temperature distributions depending on the parameters of induction heating and heat transfer conditions are presented in the form of 2D and 3D graphs.

Key words: *electroconductive panel, induction heating, quasi-steady-state electromagnetic field, near-surface and continuous heating modes, unsteady temperature field.*

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APPROXIMATE MAXIMUM LIKELIHOOD ESTIMATION FOR A TWO-THRESHOLD LÉVY PROCESS: CONVERGENCE ASSESSMENT AND DATA APPLICATIONS

This paper presents an approach to modelling complex dynamical systems using a two-threshold Lévy process, which allows for changes in process properties when crossing critical values. The proposed model combines three key components – drift, diffusion, and jump – which together enable the description of both smooth and abrupt changes in system behaviour. An important feature is the division of the process into three regimes, which enables separate parameter estimation in each the ranges. This approach enhances the flexibility of the model and makes it suitable for analysing processes with complex hierarchical dynamics. To estimate the model parameters, an algorithm based on Approximate Maximum Likelihood Estimation (AMLE) was developed. It is implemented as an iterative procedure, which refines the parameters at each step with respect to the current threshold values and continues calculations until convergence is achieved. The algorithm enables estimation of drift parameters, diffusion coefficients and jump components, as well as the identification of the optimal thresholds. Within the study, a methodology for parameter estimation across three regimes was developed, an iterative algorithm was constructed, and its software implementation was created. The algorithm was tested on a synthetically generated dataset, which made it possible to evaluate the accuracy of parameter recovery and to examine its robustness to variations in initial conditions. Particular attention was devoted to convergence analysis: numerical experiments demonstrated that, at each step of the iterative procedure, the parameter update function reduces the differences between estimates, thereby ensuring the stability of the optimisation process. The obtained results confirm the effectiveness of the proposed estimation procedure for detecting structural changes in time series. The approach not only reproduces complex process trajectories but also identifies transition points between regimes with different dynamics. In future applications, the developed methodology may be applied to the analysis of data reflecting real economic or physical processes, including financial markets, technical systems, or natural phenomena, where threshold effects and behavioural heterogeneities are observed. Thus, the two-threshold Lévy process, combined with the iterative parameter estimation algorithm, can serve as a universal tool for studying dynamical systems with multi-component structure.

Key words: *approximate maximum likelihood estimation, two-threshold diffusion process, stochastic differential equation.*

The primary aim of this article is to develop and analyse (with verification at the programming code level) a method for estimating parameters – namely, threshold values, drift, diffusion, and the jump component of a stochastic model.

As the stochastic model, a Lévy process with drift, diffusion, and jump parameters is considered [1, p. 823-841]. Analyses of different dynamic regimes of such threshold diffusion processes are presented in [2, p. 123-140]. Threshold models of diffusion processes that incorporate drift were considered in [3; 4, p. 2859-2872; 5, p. 223-262]. Models with a single threshold were discussed in [2, p. 123-140; 3]. Issues of convergence, as part of the asymptotic properties of Approximate Maximum Likelihood Estimation, were examined in [6, p. 1350-1380].

The Lévy process is defined by the Itô–Skorohod stochastic differential equation:

$$dX_t = \mu(X_t,t)\,dt + \sigma(X_t,t)\,dW_t + \int_U \gamma(X_t,t)\,\tilde{\nu}(dt,du), \quad X_0 = 0. \tag{1}$$

We now consider a stochastic process $\{X_t\}_{t\geq 0}$, that is a solution to a stochastic differential equation with two thresholds r_1 and r_2, and which includes a jump component. Numerically, this process can be expressed as:

$$\begin{gathered} dX_t = \{(\beta_{10}+\beta_{11}X_t)I(X_t\leq r_1)+(\beta_{20}+\beta_{21}X_t)I(r_1<X_t\leq r_2)+ \\ +(\beta_{30}+\beta_{31}X_t)I(X_t>r_2)\}\,dt+ \\ +\{\sigma_1 I(X_t\leq r)+\sigma_2 I(r_1<X_t\leq r_2)+\sigma_3 I(X_t>r_2)\}\,dW_t+ \\ +\int_U \{\gamma_1 I(X_t\leq r)+\gamma_2 I(r_1<X_t\leq r_2)+\gamma_3 I(X_t>r_2)\}\,\tilde{\nu}(dt,du). \end{gathered} \tag{2}$$

Where W_t denotes a Wiener process, β_{i0}, β_{i1} define the linear drift in the i-th regime ($i = 1, 2, 3$), σ_i is the diffusion coefficient in that regime, and γ_i is the jump component. The process is divided by thresholds r_1 and r_2 and into three regimes, each with distinct drift and diffusion coefficients, X_t as well as jump components.

The discretized stochastic equation under the Euler scheme is given by (3):

$$\begin{gathered}
dX_t = \Delta_j \Big\{ \left(\beta_{10} + \beta_{11} X_{j-1}\right) I\left(X_{j-1} \le r_1\right) + \left(\beta_{20} + \beta_{21} X_{j-1}\right) \times \\
\times I\left(r_1 < X_{j-1} \le r_2\right) + \left(\beta_{30} + \beta_{31} X_{j-1}\right) I\left(X_{j-1} > r_2\right) \Big\} + \\
+ \Big\{ \sigma_1 I\left(X_{j-1} \le r_1\right) + \sigma_2 I\left(r_1 < X_{j-1} \le r_2\right) + \sigma_3 I\left(X_{j-1} > r_2\right) \Big\} \left(W_j - W_{j-1}\right) + \\
\Big\{ \gamma_1 I\left(X_{j-1} \le r_1\right) + \gamma_2 I\left(r_1 < X_{j-1} \le r_2\right) + \gamma_3 I\left(X_{j-1} > r_2\right) \Big\} \Delta \nu_j .
\end{gathered} \tag{3}$$

Where $W_j - W_{j-1} \sim N\left(0,\ \Delta_j\right)$ and $\Delta_j = t_j - t_{j-1}$. If the values of r_1, r_2 and X_0 are known, one can derive the expression for the doubled negative approximate log-likelihood function (4).

$$\begin{gathered}
-2 l_X\left(\theta\right) = \\
= C + \sum_{j=1}^{q} \log \Big\{ \sigma_1^2 I\left(X_{j-1} \le r_1\right) + \sigma_2^2 I\left(r_1 < X_{j-1} \le r_2\right) + (\sigma_3^2 I\left(X_{j-1} > r_2\right) \Big\} + \\
+ \sum_{j=1}^{q} \frac{\left[X_j - X_{j-1} - \left\{ \begin{array}{c} \left(\beta_{10} + \beta_{11} X_{j-1}\right) I\left(X_{j-1} \le r_1\right) + \\ + \left(\beta_{20} + \beta_{21} X_{j-1}\right) I\left(r_1 < X_{j-1} \le r_2\right) + \\ + \left(\beta_{30} + \beta_{31} X_{j-1}\right) I\left(X_{j-1} > r_2\right) \end{array} \right\} \Delta_j \right]^2}{\Big\{ \sigma_1^2 I\left(X_{j-1} \le r_1\right) + \sigma_2^2 I\left(r_1 < X_{j-1} \le r_2\right) + (\sigma_3^2 I\left(X_{j-1} > r_2\right) \Big\} \Delta_j} - \\
-2 \sum_{j=1}^{q} X_j \log\left(\Delta_j\right) - \\
-4 \sum_{j=1}^{q} X_j \log \Big\{ \gamma_1 I\left(X_{j-1} \le r_1\right) + \gamma_2 I\left(r_1 < X_{j-1} \le r_2\right) + \gamma_3 I\left(X_{j-1} > r_2\right) \Big\} + \\
+ 2 \sum_{j=1}^{q} \Delta t_j \Big\{ \gamma_1 I\left(X_{j-1} \le r_1\right) + \gamma_2 I\left(r_1 < X_{j-1} \le r_2\right) + \gamma_3 I\left(X_{j-1} > r_2\right) \Big\}^2 .
\end{gathered} \tag{4}$$

Differentiating (4) with respect to β, σ^2 and γ and equating to 0 yields the matrix form of the system of equations for the main parameters:

$$\begin{bmatrix} \beta_{10} \\ \beta_{11} \end{bmatrix} =$$

$$= \begin{bmatrix} \sum_{i=1}^{q} \frac{\Delta_j I\left(X_{j-1} \le r_1\right)}{\left\{\sigma_1^2 I\left(X_{j-1} \le r_1\right) + \sigma_2^2 I\left(r_1 < X_{j-1} \le r_2\right) + \sigma_3^2 I\left(X_{j-1} > r_2\right)\right\}} \\ \sum_{i=1}^{q} \frac{\Delta_j I\left(X_{j-1} \le r_1\right)}{\left\{\sigma_1^2 I\left(X_{j-1} \le r_1\right) + \sigma_2^2 I\left(r_1 < X_{j-1} \le r_2\right) + \sigma_3^2 I\left(X_{j-1} > r_2\right)\right\} X_{j-1}^{-1}} \end{bmatrix.$$

$$\left. \begin{matrix} \sum_{i=1}^{q} \frac{\Delta_j I\left(X_{j-1} \le r_1\right)}{\left\{\sigma_1^2 I\left(X_{j-1} \le r_1\right) + \sigma_2^2 I\left(r_1 < X_{j-1} \le r_2\right) + \sigma_3^2 I\left(X_{j-1} > r_2\right)\right\} X_{j-1}^{-1}} \\ \sum_{i=1}^{q} \frac{\Delta_j I\left(X_{j-1} \le r_1\right)}{\left\{\sigma_1^2 I\left(X_{j-1} \le r_1\right) + \sigma_2^2 I\left(r_1 < X_{j-1} \le r_2\right) + \sigma_3^2 I\left(X_{j-1} > r_2\right)\right\} X_{j-1}^{-2}} \end{matrix} \right]^{-1} \times$$

$$\times \begin{bmatrix} \sum_{i=1}^{q} \frac{\left(X_j - X_{j-1}\right) I\left(X_{j-1} \le r_1\right)}{\left\{\sigma_1^2 I\left(X_{j-1} \le r_1\right) + \sigma_2^2 I\left(r_1 < X_{j-1} \le r_2\right) + \sigma_3^2 I\left(X_{j-1} > r_2\right)\right\}} \\ \sum_{i=1}^{q} \frac{\left(X_j - X_{j-1}\right) I\left(X_{j-1} \le r_1\right)}{\left\{\sigma_1^2 I\left(X_{j-1} \le r_1\right) + \sigma_2^2 I\left(r_1 < X_{j-1} \le r_2\right) + \sigma_3^2 I\left(X_{j-1} > r_2\right)\right\} X_{j-1}^{-1}} \end{bmatrix};$$

$$\begin{bmatrix} \beta_{20} \\ \beta_{21} \end{bmatrix} =$$

$$= \begin{bmatrix} \sum_{i=1}^{q} \frac{\Delta_j I(r_1 < X_{j-1} \le r_2)}{\left\{\sigma_1^2 I\left(X_{j-1} \le r_1\right) + \sigma_2^2 I\left(r_1 < X_{j-1} \le r_2\right) + \sigma_3^2 I\left(X_{j-1} > r_2\right)\right\}} \\ \sum_{i=1}^{q} \frac{\Delta_j I(r_1 < X_{j-1} \le r_2)}{\left\{\sigma_1^2 I\left(X_{j-1} \le r_1\right) + \sigma_2^2 I\left(r_1 < X_{j-1} \le r_2\right) + \sigma_3^2 I\left(X_{j-1} > r_2\right)\right\} X_{j-1}^{-1}} \end{bmatrix.$$

$$\left. \begin{matrix} \sum_{i=1}^{q} \frac{\Delta_j I(r_1 < X_{j-1} \le r_2)}{\left\{\sigma_1^2 I\left(X_{j-1} \le r_1\right) + \sigma_2^2 I\left(r_1 < X_{j-1} \le r_2\right) + \sigma_3^2 I\left(X_{j-1} > r_2\right)\right\} X_{j-1}^{-1}} \\ \sum_{i=1}^{q} \frac{\Delta_j I(r_1 < X_{j-1} \le r_2)}{\left\{\sigma_1^2 I\left(X_{j-1} \le r_1\right) + \sigma_2^2 I\left(r_1 < X_{j-1} \le r_2\right) + \sigma_3^2 I\left(X_{j-1} > r_2\right)\right\} X_{j-1}^{-2}} \end{matrix} \right]^{-1} \times$$

$$\times\left[\begin{array}{c}\sum_{i=1}^{q}\frac{\left(X_j-X_{j-1}\right)I(r_1<X_{j-1}\le r_2)}{\left\{\sigma_1^2 I\left(X_{j-1}\le r_1\right)+\sigma_2^2 I\left(r_1<X_{j-1}\le r_2\right)+\sigma_3^2 I\left(X_{j-1}>r_2\right)\right\}}\\ \sum_{i=1}^{q}\frac{\left(X_j-X_{j-1}\right)I(r_1<X_{j-1}\le r_2)}{\left\{\sigma_1^2 I\left(X_{j-1}\le r_1\right)+\sigma_2^2 I\left(r_1<X_{j-1}\le r_2\right)+\sigma_3^2 I\left(X_{j-1}>r_2\right)\right\}X_{j-1}^{-1}}\end{array}\right];$$

$$\begin{bmatrix}\beta_{30}\\ \beta_{31}\end{bmatrix}=$$

$$=\left[\begin{array}{cc}\sum_{i=1}^{q}\frac{\Delta_j I(X_{j-1}>r_2)}{\left\{\sigma_1^2 I\left(X_{j-1}\le r_1\right)+\sigma_2^2 I\left(r_1<X_{j-1}\le r_2\right)+\sigma_3^2 I\left(X_{j-1}>r_2\right)\right\}} & \sum_{i=1}^{q}\frac{\Delta_j I(X_{j-1}>r_2)}{\left\{\sigma_1^2 I\left(X_{j-1}\le r_1\right)+\sigma_2^2 I\left(r_1<X_{j-1}\le r_2\right)+\sigma_3^2 I\left(X_{j-1}>r_2\right)\right\}X_{j-1}^{-1}}\\ \sum_{i=1}^{q}\frac{\Delta_j I(X_{j-1}>r_2)}{\left\{\sigma_1^2 I\left(X_{j-1}\le r_1\right)+\sigma_2^2 I\left(r_1<X_{j-1}\le r_2\right)+\sigma_3^2 I\left(X_{j-1}>r_2\right)\right\}X_{j-1}^{-1}} & \sum_{i=1}^{q}\frac{\Delta_j I(X_{j-1}>r_2)}{\left\{\sigma_1^2 I\left(X_{j-1}\le r_1\right)+\sigma_2^2 I\left(r_1<X_{j-1}\le r_2\right)+\sigma_3^2 I\left(X_{j-1}>r_2\right)\right\}X_{j-1}^{-2}}\end{array}\right]^{-1}\times$$

$$\times\left[\begin{array}{c}\sum_{i=1}^{q}\frac{\left(X_j-X_{j-1}\right)I(X_{j-1}>r_2)}{\left\{\sigma_1^2 I\left(X_{j-1}\le r_1\right)+\sigma_2^2 I\left(r_1<X_{j-1}\le r_2\right)+\sigma_3^2 I\left(X_{j-1}>r_2\right)\right\}}\\ \sum_{i=1}^{q}\frac{\left(X_j-X_{j-1}\right)I(X_{j-1}>r_2)}{\left\{\sigma_1^2 I\left(X_{j-1}\le r_1\right)+\sigma_2^2 I\left(r_1<X_{j-1}\le r_2\right)+\sigma_3^2 I\left(X_{j-1}>r_2\right)\right\}X_{j-1}^{-1}}\end{array}\right];$$

$$\sigma_1^2=\frac{\sum_{i=1}^{q}\frac{\left\{X_j-X_{j-1}-\Delta_j\left(\beta_{10}+\beta_{11}X_{j-1}\right)\right\}^2 I\left(X_{j-1}\le r_1\right)}{\left\{\sigma_1^4 I\left(X_{j-1}\le r_1\right)+\sigma_2^4 I\left(r_1<X_{j-1}\le r_2\right)+\sigma_3^4 I\left(X_{j-1}>r_2\right)\right\}\Delta_j}}{\sum_{i=1}^{q}\frac{I\left(X_{j-1}\le r_1\right)}{\left\{\sigma_1^2 I\left(X_{j-1}\le r_1\right)\right\}}};$$

$$\sigma_2^2=\frac{\sum_{i=1}^{q}\frac{\left\{X_j-X_{j-1}-\Delta_j\left(\beta_{20}+\beta_{21}X_{j-1}\right)\right\}^2 I(r_1<X_{j-1}\le r_2)}{\left\{\sigma_1^4 I\left(X_{j-1}\le r_1\right)+\sigma_2^4 I\left(r_1<X_{j-1}\le r_2\right)+\sigma_3^4 I\left(X_{j-1}>r_2\right)\right\}\Delta_j}}{\sum_{i=1}^{q}\frac{I(r_1<X_{j-1}\le r_2)}{\left\{\sigma_2^2 I\left(r_1<X_{j-1}\le r_2\right)\right\}}};$$

$$\sigma_3^2 = \frac{\sum_{i=1}^{q} \frac{\left\{X_j - X_{j-1} - \Delta_j\left(\beta_{30} + \beta_{31}X_{j-1}\right)\right\}^2 I\left(X_{j-1} > r_2\right)}{\left\{\sigma_1^4 I\left(X_{j-1} \le r_1\right) + \sigma_2^4 I\left(r_2 > X_{j-1} \ge r_1\right) + \sigma_3^4 I\left(X_{j-1} > r_2\right)\right\}\Delta_j}}{\sum_{i=1}^{q} \frac{I\left(X_{j-1} > r_2\right)}{\left\{\sigma_3^2 I\left(X_{j-1} > r_2\right)\right\}}}$$

$$\gamma_1 = \frac{\sum_{j=1}^{q} \frac{X_j I\left(X_{j-1} \le r_1\right)}{\gamma_1 I\left(X_{j-1} \le r_1\right) + \gamma_2 I\left(r_2 > X_{j-1} \ge r_1\right) + \gamma_1 I\left(X_{j-1} > r_2\right)}}{\sum_{j=1}^{q} \Delta\nu_j I\left(X_{j-1} \le r_1\right)}$$

$$\gamma_2 = \frac{\sum_{j=1}^{q} \frac{(X_j I(r_2 > X_{j-1} \ge r_1))}{\gamma_1 I\left(X_{j-1} \le r_1\right) + \gamma_2 I\left(r_2 > X_{j-1} \ge r_1\right) + \gamma_1 I\left(X_{j-1} > r_2\right)}}{\sum_{j=1}^{q} \Delta\nu_j I(r_2 > X_{j-1} \ge r_1)}$$

$$\gamma_3 = \frac{\sum_{j=1}^{q} \frac{(X_j I(X_{j-1} > r_2))}{\gamma_1 I\left(X_{j-1} \le r_1\right) + \gamma_2 I\left(r_2 > X_{j-1} \ge r_1\right) + \gamma_1 I\left(X_{j-1} > r_2\right)}}{\sum_{j=1}^{q} \Delta\nu_j I(X_{j-1} > r_2)} \qquad (5)$$

Computational Algorithm. Based on the system of equations (5).

Step 1. Dataset for the model.

A time series $\left\{X_0, X_1, \ldots, X_q\right\}$, is given, for which the order statistics are computed, where $\left\{X_{(0)}, X_{(1)}, \ldots, X_{(q)}\right\}$, where $X_{(0)} \le X_{(1)} \le \ldots \le X_{(q)}$.

Step 2.

2.1. Fix $\hat{r}$ for $i = 0, 1, \ldots, \lambda$**.** Define

$$\hat{\sigma}_1^2(0), \hat{\sigma}_2^2(0), \hat{\sigma}_3^2(0), \hat{\gamma}_1(0), \hat{\gamma}_2(0), \hat{\gamma}_3(0).$$

2.2. For $k \ge 1$ compute

$$\theta(k) = \left(\hat{\beta}_{10}(k), \hat{\beta}_{11}(k), \hat{\beta}_{20}(k), \hat{\beta}_{21}(k), \hat{\beta}_{30}(k),\right.$$

$$\left.\hat{\beta}_{31}(k), \hat{\sigma}_1^2(k), \hat{\sigma}_2^2(k), \hat{\sigma}_3^2(k), \hat{\gamma}_1(k), \hat{\gamma}_2(k), \hat{\gamma}_3(k)\right),$$

recursively, as in equations (5), substituting into the right-hand sides the parameter values $(k-1)$ obtained at the iteration step.

2.3. Repeat Step 2.2 until $\hat{\theta}(k)$ convergence is achieved.

Step 3.

For $i = 0,1,\dots,\lambda$ compute $-2l_X\left(\hat{\theta}_i\right)$. Then $\tau = \text{argmin}_i\left\{-2l_X\left(\hat{\theta}_i\right)\right\}$ and the approximate maximum likelihood estimate is $= (r,\theta)$ $\hat{\eta} = \left(\hat{r}_\tau, \hat{\theta}_\tau\right)$.

Numerical Convergence of the Iterative Algorithm. In estimating the parameters of a model with two thresholds r_1 and r_2, which partition the observations into three regions, an iterative procedure is employed to calculate the model parameters: β_{i0}, β_{i1} for $i = 1,2,3$ and $\sigma_1^2, \sigma_2^2, \sigma_3^2$. and $\gamma_1, \gamma_2, \gamma_3$

The vector (set) of model parameters has the form:

$$\theta = \left(\beta_{10}, \beta_{11}, \beta_{20}, \beta_{21}, \beta_{30}, \beta_{31}, \sigma_1^2, \sigma_2^2, \sigma_3^2, \gamma_1, \gamma_2, \gamma_3\right).$$

At the k iteration, the algorithm updates the parameters as follows

$$\theta^{(k+1)} = F\left(\theta^{(k)}\right),$$

where F is a function used to compute new parameters based on current values.

Banach's Theorem states that if the function F reduces the difference between successive values at each iteration, then the process converges to a stable result, i.e.,

$$F\left(\theta^{(1)}\right) - F\left(\theta^{(2)}\right) \le q\theta^{(1)} - \theta^{(2)},\ 0 < q < 1,$$

where a unique fixed point θ^* exists, and the sequence $\theta^{(k)}$ converges to it. To verify this property, spectral analysis of the Jacobian matrix can be used.

The Jacobian Matrix $J(\theta)$ is a set of partial derivatives:

$$J_{ij} = \frac{\partial F_i(\theta)}{\partial \theta_j},$$

meaning that each element indicates how parameter θ_i changes with respect to parameter θ_j.

For a two-threshold model, the Jacobian matrix has dimension 10×10, for example:

$$J = \begin{pmatrix} \dfrac{\partial \beta_{10}^{(k+1)}}{\partial \beta_{10}^{(k)}} & \cdots & \dfrac{\partial \beta_{10}^{(k+1)}}{\partial \sigma_3^{2,(k)}} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial \sigma_3^{2,(k+1)}}{\partial \beta_{10}^{(k)}} & \cdots & \dfrac{\partial \gamma^{k+1}}{\partial \gamma^k} \end{pmatrix}$$

To evaluate the Jacobian matrix at each step of the iterative algorithm, instead of analytical derivatives, a numerical approximation method was employed based on a one-sided finite-difference scheme (forward difference):

$$J_{ij} \approx \frac{\theta_i^{(k+1)} - \theta_i^{(k)}}{\theta_j^{(k)}} \left(\text{for } \theta_j^{(k)} \neq 0 \right).$$

Here, the numerator represents the change in the value of θ_i between iterations, and the denominator is the previous value of parameter θ_j, thereby approximating the relative change of one parameter with respect to another.

After constructing the approximate Jacobian matrix *J*, its spectral radius – the largest eigenvalue modulus among all eigenvalues of $\lambda_1, \lambda_2, \ldots, \lambda_n$: – is computed:

$$\rho(J) = max|\lambda_i|.$$

A correct convergence assessment ultimately affects both the number of iterations required and the accuracy of parameter estimation. During algorithm implementation, several factors influencing the iterative procedure were considered: initial threshold values, stopping criteria, and evaluation of parameter changes between iterations.

Numerical analysis was employed to substantiate convergence, allowing verification that at each step the parameter update function makes the values progressively closer. This approach provides a mathematical justification of the viability of the proposed model.

Evaluation of Algorithm Performance. To verify the algorithm's performance, program code was developed in the RStudio environment, implementing the above-described procedure and enabling calculation of parameters $\beta_{10}, \beta_{11}, \beta_{20}, \beta_{21}, \beta_{30}, \beta_{31}, \sigma_1^2, \sigma_2^2, \sigma_3^2, \gamma_1, \gamma_2, \gamma_3$, as well as threshold values and r_1, r_2.

To assess the efficiency and convergence of the estimation procedure, a synthetic time series of length *n* = 105 modeling a stochastic process with three distinct dynamic regimes. The constructed series included: a decreasing trend in the first segment $n_1 = 25$ 25 observations), an almost stationary trend in the second segment ($n_2 = 35$) and an increasing trend in the final segment ($n_3 = 45$). Each regime was characterized by a linear deterministic drift component superimposed with Gaussian noise, whose variance varied across regimes, as well as jump components. These regimes were combined to form the complete dataset $\{X_t\}_{t=1}^n$, designed to represent a process transitioning through structural thresholds with different statistical properties.

During implementation, the algorithm successfully estimated the main drift coefficients β_{ij}, diffusion parameters $\hat{\sigma}_i^2$, and jump components γ_i, corresponding to each regime. The calculated threshold values $\hat{r}_1$ and $\hat{r}_2$ precisely matched the true transition points embedded in the synthetic data generation process, thereby confirming the ability of the estimation procedure to detect transition points between different dynamic behaviours (Fig 1).

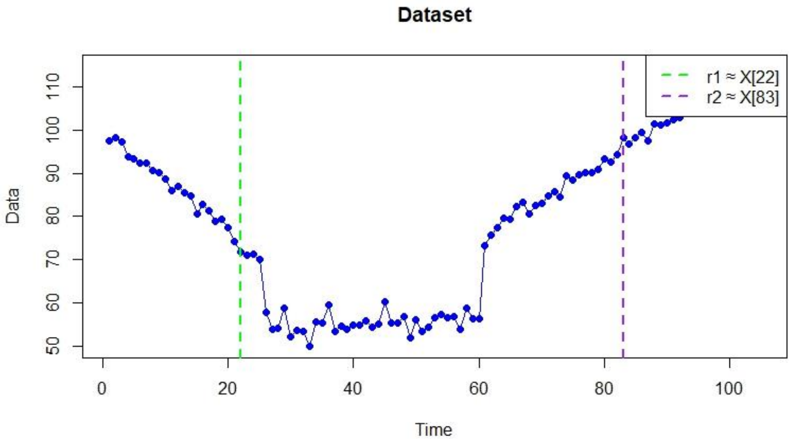

***Fig. 1.** The thresholds $r_1 \approx X[22]$ and $r_1 \approx X[83]$, n_1 is decreasing,*
n_2 is almost stationary, and n_3 is increasing

Conclusions. The methodology developed in this study enables not only the estimation of drift, diffusion, and jump parameters for the regimes of the Ornstein-Uhlenbeck process, but also the identification of threshold points r_1 and r_2 which partition the process dynamics into distinct segments with different properties. Implementation of the iterative algorithm in program code confirmed the correctness of the computations and demonstrated sensitivity to structural changes in the process data.

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НАБЛИЖЕНИЙ МЕТОД МАКСИМАЛЬНОЇ ВІРОГІДНОСТІ ДЛЯ ОЦІНЮВАННЯ ДВОПОРОГОВОГО ПРОЦЕСУ ЛЕВІ. ОЦІНКА ЗБІЖНОСТІ ТА РОБОТА З ДАНИМИ

У роботі представлено підхід до моделювання складних динамічних систем із використанням двопорогового процесу Леві, що дозволяє враховувати зміну властивостей процесу при переході через критичні значення. Запропонована модель поєднує три ключові компоненти: зсув, дифузію та стрибкову складову, які разом забезпечують опис як плавних, так і різких змін у поведінці системи. Важливою характеристикою є поділ процесу на три режими, що надає змогу окремо оцінювати параметри в кожному з діапазонів. Такий підхід підвищує гнучкість моделі та робить її придатною для аналізу процесів зі складною ієрархічною динамікою. Для оцінювання параметрів моделі розроблено алгоритм, що базується на наближеній оцінці максимальної правдоподібності. Він реалізований у вигляді ітераційної процедури, яка на кожному кроці уточнює параметри з урахуванням поточних порогових значень і продовжує обчислення до досягнення збіжності. Алгоритм дозволяє оцінювати параметри зсуву, дифузії та стрибкової компоненти, а також здійснює пошук оптимальних порогів. У рамках дослідження було розроблено методику оцінки параметрів у трьох режимах, побудовано ітераційний алгоритм та створено його програмну реалізацію. Алгоритм перевірявся на синтетично згенерованому наборі даних, що дало змогу оцінити точність відновлення параметрів і дослідити його стійкість до варіацій початкових умов. Окрему увагу приділено аналізу збіжності: за допомогою чисельних експериментів продемонстровано, що на кожному кроці ітераційної процедури функція оновлення параметрів зменшує різницю між їхніми оцінками, що гарантує стабільність процесу оптимізації. Отримані результати підтверджують ефективність запропонованого методу для задач виявлення структурних змін у часових рядах. Запропонований підхід дозволяє не лише відтворювати складні траєкторії процесів, а й локалізувати моменти переходу між режимами з різною динамікою. У перспективі розроблена методика може бути застосована до аналізу даних, що відображають реальні економічні чи фізичні процеси, зокрема фінансових ринків, технічних систем або природних явищ, де спостерігаються порогові зміни та неоднорідності у поведінці. Таким чином, модель двопорогового процесу Леві у поєднанні з ітераційним алгоритмом оцінки параметрів може слугувати універсальним інструментом для дослідження динамічних систем із багатокомпонентною структурою.

Ключові слова: *наближений метод максимальної правдоподібності, двопороговий дифузійний процес, стохастичне диференціальне рівняння.*

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