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Sachovska V. A.

ORCID: 0009-0004-1046-7242,

PhD Student, National University of Ostroh Academy, Ostroh, Ukraine,

E-mail: vitalina.sachovska@oa.edu.ua

Pertsov A. S.

ORCID: 0000-0002-2434-3652,

PhD in Physical and Mathematical Sciences, Yuriy Fedkovych

Chernivtsi National University, Chernivtsi, Ukraine,

E-mail: a.pertsov@chnu.edu.ua

STOCHASTIC MODELING OF TECHNOLOGICAL LABOR MARKET DYNAMICS WITH FAST MARKOV SWITCHING VIA THE AVERAGING PRINCIPLE

This study considers a stochastic evolutionary representation of the technological labor market, focusing on the interaction between employment and automation in the IT sector. The dynamics are described by a nonlinear competition system of Lotka-Volterra type, extended by a rapidly switching random environment modeled through a Markov process. Such a formulation makes it possible to reflect changes in technological regimes and their influence on the structure of the labor market beyond the limitations of deterministic models.

The presence of fast stochastic switching leads to analytical difficulties, which are addressed by employing an averaging approach. Assuming ergodicity of the underlying Markov process, the original stochastic system can be replaced, in the limit, by a deterministic model whose coefficients are obtained from the stationary distribution of the environment. This reduction preserves the qualitative features of the system while making its analysis more tractable.

The behavior of the model is illustrated through several scenarios reflecting different technological environments, including innovative adaptation, technological polarization, and unstable regime switching. The simulation results show that, in most cases, stochastic trajectories remain concentrated near the corresponding averaged dynamics, confirming the applicability of the averaging approach. At the same time, noticeable deviations may arise in asymmetric environments with strong nonlinear effects. The developed framework can be used to investigate long-term interactions

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between automation and employment and to assess structural changes in the technological labor market under uncertainty.

Key words: *stochastic systems, averaging method, Markov switching, technology labor market.*

Introduction. The expansion of artificial intelligence technologies is reshaping the structure of contemporary labor markets, particularly within high-technology sectors. In the IT domain, automated systems increasingly perform tasks that were previously associated with human expertise, including complex cognitive operations. As a result, both the composition of employment and the set of required professional skills undergo significant transformation [1, 3, 5]. In contrast to earlier stages of automation, current technologies extend their influence beyond routine activities, affecting a broad spectrum of intellectual work [4, 7]. This evolution raises a number of economic and societal challenges related to workforce adaptation, technological competitiveness, and the long-term sustainability of labor market development.

The relationship between technological advancement and employment is frequently interpreted as an interaction between human labor and automated systems. Such interaction can be represented within the framework of competitive dynamics, where both components influence each other over time [2, 9]. Mathematical descriptions of this type are often based on classical competition models originating from the works of Volterra [10]. However, contemporary empirical evidence and theoretical studies indicate that this interaction has become considerably more complex, requiring the use of extended modeling approaches [6, 8]. In particular, deterministic formulations are unable to capture random fluctuations of the environment and abrupt technological shifts, which are essential features of real-world labor market dynamics.

To overcome these limitations, recent studies rely on stochastic modeling frameworks that incorporate random perturbations and regime-switching mechanisms [11-18, 20-26]. When the underlying environment evolves on a fast time scale, the analysis can be facilitated by applying averaging-based techniques [13, 19]. In this setting, as the parameter $\varepsilon \rightarrow 0$, the stochastic trajectories approach those of a deterministic system with coefficients determined by the stationary distribution of the switching process. Although these methods are well established in the theory of stochastic systems, their application to technological labor market modeling, especially in the context of AI-driven transformations, remains relatively limited.

The present study aims to develop a mathematical model describing the interaction between employment and automation in the technological labor market. The proposed framework combines a competition-based structure of the Volterra type with a stochastic representation of the technological environment in the form of a Markov process. The use of aver-

aging techniques allows one to derive an effective deterministic system that captures the long-term behavior of the original stochastic model.

Baseline Model of the Labor Market. To model the interplay between human labor and automation, we use a reduced predator-prey type representation [10]:

$$\begin{cases} \frac{dN(t)}{dt} = aN(t) - bN(t)A(t) - cN^2(t), \\ \frac{dA(t)}{dt} = -dA(t) + eN(t)A(t), \quad t > 0, \end{cases} \quad (1)$$

where $N(t)$ – the workforce size in the IT sector at time t ; $A(t)$ – the level of automation at time t , quantified through the intensity of AI-based technologies in the IT sector; a – the rate of workforce expansion in the IT sector due to growing demand for skilled labor; b – the substitution effect of automation, reflecting a reduction in demand for IT specialists; cN^2 – the intensification of job competition driven by the growth of the IT workforce; dA – the natural decay of automation due to technological obsolescence and ongoing maintenance requirements; eN – reflects automation growth enabled by the integration of human expertise and AI technologies.

Averaging Method. An evolutionary system with random perturbations in the small-parameter $\varepsilon \rightarrow 0 (\varepsilon > 0)$ series scheme is defined by [20]:

$$\frac{du^\varepsilon(t)}{dt} = C\left(u^\varepsilon(t), x\left(\frac{t}{\varepsilon}\right)\right), t \geq 0, \quad (2)$$

where the intensity function $C(u; x) u \in R^d, x \in E$, is a vector-valued function of two arguments: the argument of the evolutionary system u , which takes values in the space $R^d, d \geq 1$, and the random perturbation argument x , which takes values in the phase space (X, \mathcal{F}) , which is a complete metric space X with a measurable sets of \mathcal{F} . We assume that individual points x of the space X belong to \mathcal{F} .

Here $x\left(\frac{t}{\varepsilon}\right)$ indicates that the random perturbation x evolves on a much faster time scale than the underlying system x . This leads to a high-frequency noise effect, which, in the limit, is replaced by its averaged impact.

It is assumed that a solution to the family of equations exists:

$$\frac{du_x(t)}{dt} = C(u_x(t), x), x \in X. \quad (3)$$

A key property of equations (2) is the semigroup property of the solutions:

$$u_x(t_1 + t_2; u) = u_x(t_2; u_x(t_1; u)). \quad (4)$$

Here $u_x(t; u)$ is the solution to equation (3) with the initial condition $u_x(0; u) = u$.

The semigroup property (4) induces a semigroup of operators in a Banach space of test functions $\mathbb{B}(R^d)$:

$$C_t(x)\varphi(u) = \varphi(u_x(t; u)), t \geq 0, \varphi(u) \in \mathbb{B}(R^d). \quad (5)$$

The semigroup of operators (5) is determined by its generator:

$$C(x)\varphi(u) := \lim_{t \rightarrow 0} \frac{1}{t} [C_t(x)\varphi(u) - \varphi(u)],$$

which is given by the formula

$$C(x)\varphi(u) = C(u; x)\varphi'(u).$$

By definition, we obtain

$$C(u; x)\varphi'(u) = \sum_{k=1}^d C_k(u; x) \frac{\partial \varphi(u)}{\partial u_k}.$$

We now turn to the characterization of random perturbations. In what follows, we mainly consider random perturbations governed by a Markov jump process $x(t), t \geq 0$, in the standard phase space (X, \mathcal{F}) , which is specified by the infinitesimal generator in a Banach space $\mathbb{B}(E)$ test functions $\varphi(x) \in \mathbb{B}(E)$, which are real-valued:

$$Q\varphi(x) = q(x) \int_E P(x, dy) [\varphi(y) - \varphi(x)].$$

Here function $q(x), x \in E$, defines the jump intensity of the process $x(t), t \geq 0$, which follow an exponential distribution; stochastic kernel $P(x, dy), x \in E, dy \in \mathcal{F}$, specifies the transition probability distribution of the process $x(t)$ from state x to a subset of states dy .

Stochastic kernel $P(x, B), x \in E, B \in \mathcal{F}$ determines a uniformly ergodic embedded Markov chain $x_n = x(\tau_n), n \geq 0$, with a stationary distribution $\rho(B), B \in \mathcal{F}$. Stationary distribution $\pi(B), B \in \mathcal{F}$, Markov process $x(t), t \geq 0$, is defined by the relation:

$$\pi(dx)q(x) = q\rho(dx), q = \int_X \pi(dx)q(x).$$

Averaging Principle. Suppose that the Markov perturbation process $x(t), t \geq 0$, is uniformly ergodic and possesses a stationary distribution $\pi(B), B \in \mathcal{F}$, then, under the conditions formulated above, weak convergence takes place:

$$u^\varepsilon(t) \rightarrow u^0(t), \varepsilon \rightarrow 0.$$

Limit evolution $u^0(t), t \geq 0$, is determined by the solution of a deterministic evolutionary equation

$$\frac{du^0(t)}{dt} = \hat{C}(u^0(t)), t \geq 0,$$

where the averaged rate function is given by the relation

$$\hat{C}(u) = \int_E \pi(dx)C(u; x).$$

In our study, the state vector takes the form $u(t) = (N(t), A(t))$, where $N(t)$ – the number of employees in the IT sector; $A(t)$ – and the level of automation. Then, the rate function $C(u; x)$ is given by a Lotka-Volterra system [10]:

$$C(u; x) = \begin{pmatrix} a - cN & -bN \\ eA & -d \end{pmatrix} \times \begin{pmatrix} N \\ A \end{pmatrix},$$

here, the parameters a, b, c, e, d are depend on the state of the fast Markov environment, which describes changes in technological regimes. According to the averaging principle, as $\varepsilon \rightarrow 0$ the stochastic system (2) is asymptotically replaced by a deterministic system (1), where the averaged coefficients are defined with respect to the stationary distribution π :

$$\hat{a} = \int_E \pi(dx)a(x), \hat{b} = \int_E \pi(dx)b(x), \hat{c} = \int_E \pi(dx)c(x),$$

$$\hat{d} = \int_E \pi(dx)d(x), \hat{e} = \int_E \pi(dx)e(x).$$

As a result, the impact of rapid random switching is effectively replaced by its mean effect, yielding a deterministic system that governs the long-term dynamics of the labor market.

Analysis of Numerical Simulation Results. The numerical experiment is based on Monte Carlo simulation, where multiple realizations of

the stochastic system are generated and then averaged. This allows us to compare three different descriptions of the system: the deterministic trajectory, the stochastic trajectory, and the solution of the averaged model.

In the innovative adaptation scenario, the system is analyzed under conditions where technological progress is accompanied by gradual labor market adjustment. The deterministic model is defined by the parameters $a = 0.56, c = 10^{-4}c = 10 - 4, d = 0.49, e = 0.019$, reflecting moderate workforce growth, relatively weak substitution effects, and a balanced interaction between labor and automation. The stochastic version of the model assumes that these parameters vary depending on the state of a fast Markov environment. The corresponding stationary distribution $\pi = (0.031, 0.095, 0.238, 0.635)$ indicates that the system spends most of the time in the regime associated with more advanced automation. Averaging with respect to this distribution yields effective parameters $\hat{a} = 0.550, b = 0.0230, \hat{c} = 1.07 \cdot 10^{-4}, \hat{d} = 0.470, \hat{e} = 0.0210$, which are close to the deterministic ones but slightly shifted toward stronger automation effects.

The time dynamics shown in Fig. 1 demonstrate a strong agreement between the averaged model and the mean stochastic trajectory. Despite the presence of random switching, the individual stochastic paths form a relatively tight bundle around the averaged curve. This indicates that the effect of fast switching is effectively «smoothed out» over time. The deterministic trajectory, however, exhibits a noticeable phase shift and slightly different amplitude. This discrepancy arises because the deterministic model does not account for the redistribution of time across different regimes encoded in π , whereas the averaged model does. In other words, even though the parameters of the deterministic system are close to the averaged ones, the nonlinear structure of the model amplifies small differences, leading to visible deviations.

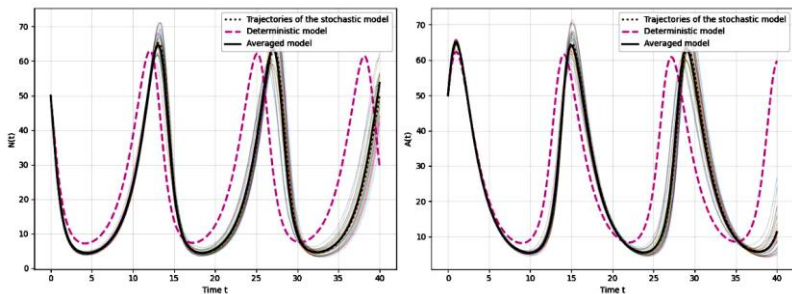


Fig. 1. Innovative Adaptation: system dynamics of $N(t)$ and $A(t)$

The distribution of final values presented in Fig. 2 further supports this observation. The Monte Carlo histogram for $N(T)$ is centered around

52, while for $A(T)$ it is concentrated near 12. The averaged model produces values $N(T) = 53.68$ and $A(T) = 11.30$, which lie well within the core of the stochastic distributions. The spread of the histograms remains moderate, indicating that the system is relatively stable under random perturbations. At the same time, a slight asymmetry of the distributions can be observed, especially for $A(T)$, suggesting that occasional visits to high-automation states have a cumulative effect on the system.

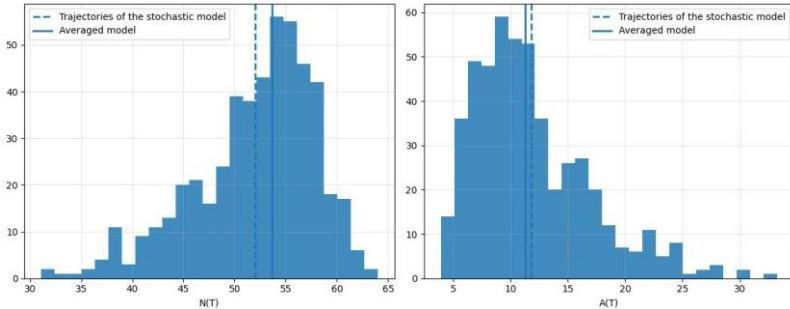


Fig. 2. *Innovative Adaptation: Distribution of terminal values of the stochastic model*

Overall, this scenario illustrates a regime in which the averaging principle works particularly well: the stochastic system behaves in a way that is close to its averaged counterpart, and the uncertainty introduced by random switching does not significantly distort the long-term dynamics.

As the next scenario we consider the technological polarization, here the system evolves under conditions where automation intensifies unevenly across regimes. The deterministic parameters:

$$a = 0.52, b = 0.024, c = 1.1 \cdot 10^{-4}, d = 0.47, e = 0.0185.$$

Averaged parameters $\hat{a} = 0.4587$, $\hat{b} = 0.0323$, $\hat{c} = 1.35 \cdot 10^{-4}$, $\hat{d} = 0.4286$, $\hat{e} = 0.0244$ are transformed with respect to the stationary distribution $\pi = (0.006, 0.049, 0.270, 0.676)$, which is strongly concentrated in high-automation states. As a result, the effective parameters shift significantly toward stronger automation effects, in particular through higher values \hat{b} and \hat{e} . Monte Carlo simulations reveal a noticeable discrepancy between the averaged and stochastic dynamics. As shown in Fig. 3, the averaged trajectory underestimates the workforce level and overestimates automation compared to the mean stochastic path.

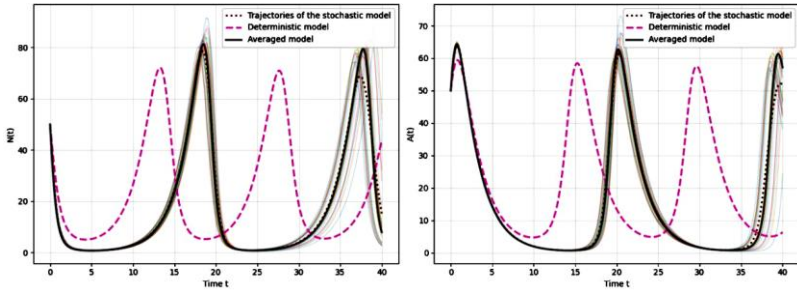


Fig. 3. Technological polarization: system dynamics of $N(t)$ and $A(t)$

The distributions in Fig. 4 confirm this behavior. The stochastic outcomes are widely spread, with $N(t)$ often remaining higher than predicted by the averaged model, while $A(t)$ exhibits a strong rightward shift. The averaged solution lies within the distribution but closer to its boundary, indicating a systematic bias. This scenario highlights that when the Markov environment is highly asymmetric, averaging may no longer fully capture the actual system dynamics.

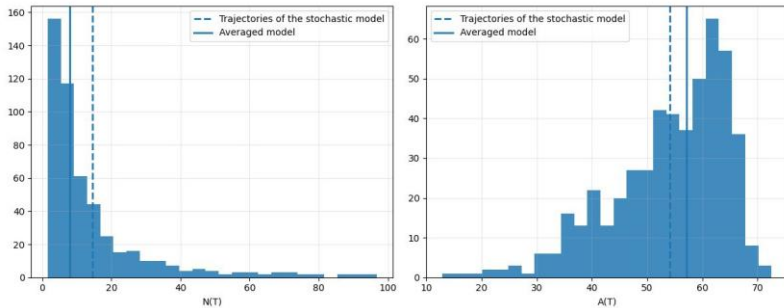


Fig. 4. Technological polarization: Distribution of terminal values of the stochastic model

The unstable technological environment scenario considers a setting with frequent switching between technological regimes. The deterministic parameters $a = 0.55$, $b = 0.0225$, $c = 1.05 \cdot 10^{-4}$, $d = 0.485$, $e = 0.0182$. Averaged parameters $\hat{a} = 0.5307$, $\hat{b} = 0.02403$, $\hat{c} = 1.11 \cdot 10^{-4}$, $\hat{d} = 0.4716$, $\hat{e} = 0.0191$ are transformed using a relatively balanced stationary distribution $\pi = (0.087, 0.232, 0.331, 0.351)$ which leads to moderate adjustments of the coefficients without strong bias toward any single regime. Monte Carlo simulations show that, despite visible variability in

individual trajectories, the stochastic dynamics closely follow the averaged model (Fig. 5). This indicates that rapid switching effectively smooths out the influence of individual regimes, making the averaging approximation reliable in this case. The deterministic trajectory, as before, deviates slightly due to the absence of regime mixing.

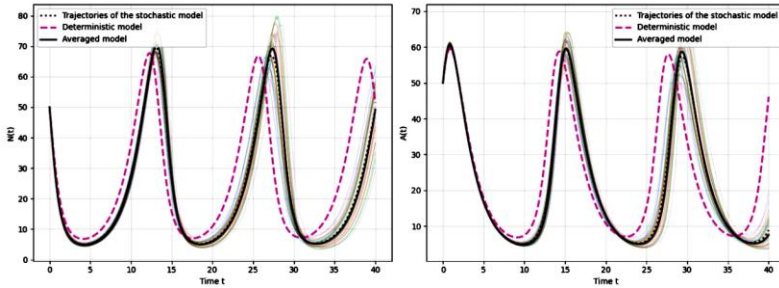


Fig. 5. Unstable technological environment: system dynamics of $N(t)$ and $A(t)$

The distributions in Fig. 6 are wider than in the first scenario, reflecting increased variability, but remain centered near the averaged solution. Both $N(t)$ and $A(t)$ show no strong skewness, suggesting that no single regime dominates the long-term outcome. Overall, this scenario confirms that when switching is sufficiently fast and balanced, the averaged model provides an accurate description of the system’s behavior.

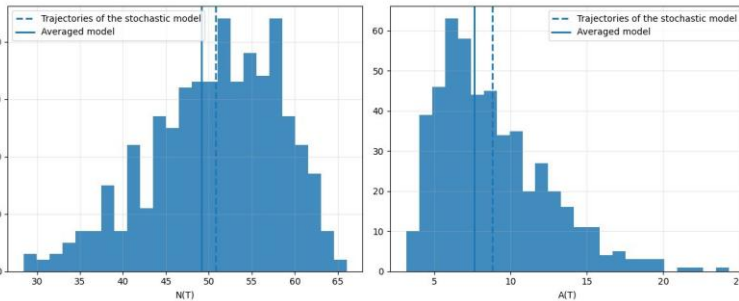


Fig. 6. Unstable technological environment: Distribution of terminal values of the stochastic model

Conclusions. The study introduces a mathematical framework for describing the interaction between employment and automation within the technological labor market. The model is constructed using a competition-based structure of the Volterra type and is extended by incorporating stochastic variations of the technological environment represented by a Markov process. By employing an averaging-based transformation, the original stochastic formulation is reduced to an effective deterministic system

with parameters defined through the stationary distribution of the switching process. This representation retains the main qualitative properties of the system while making its analysis more accessible.

The simulation results reveal that the trajectories of the stochastic system remain closely aligned with the corresponding averaged dynamics, which provides numerical evidence supporting the theoretical assumptions associated with fast-switching environments. This behavior confirms that the applied approach adequately captures the long-term tendencies of the system.

The analysis of different scenarios demonstrates distinct patterns of system evolution. A regime of innovative adaptation leads to a balanced interaction between labor and automation, where the system remains relatively stable and fluctuations are limited. In contrast, technological polarization results in a pronounced shift toward automation-dominated states, accompanied by a reduction in workforce levels and increased variability of outcomes. Finally, in an unstable technological environment with frequent regime switching, the system exhibits higher variability, while its average behavior remains close to the dynamics predicted by the averaged model. These results highlight that the long-term evolution of the labor market is strongly influenced not only by the level of automation itself but also by the structure and stability of the underlying technological environment.

In general, the proposed modeling framework can be considered as a useful tool for investigating structural changes in the labor market under ongoing technological transformation. The model may be further developed by incorporating additional stochastic factors, heterogeneous agent structures, or alternative forms of interaction between labor and technology.

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СТОХАСТИЧНЕ МОДЕЛЮВАННЯ ДИНАМІКИ ТЕХНОЛОГІЧНОГО РИНКУ ПРАЦІ ЗІ ШВИДКИМИ МАРКОВСЬКИМИ ПЕРЕМИКАННЯМИ НА ОСНОВІ ПРИНЦИПУ УСЕРЕДНЕННЯ

У роботі розглянуто стохастичну еволюційну модель технологічного ринку праці, зосереджену на взаємодії між зайнятістю та автоматизацією в ІТ-секторі. Динаміка системи описується нелінійною конкурентною моделлю типу Лотки-Вольтерри, розширеною швидкозмінним випадковим середовищем, представленим марковським процесом. Така постановка дозволяє врахувати зміну технологічних режимів та їх вплив на структуру ринку праці, виходячи за межі можливостей детермінованих моделей.

Наявність швидких стохастичних перемикань ускладнює аналітичне дослідження системи, що долається шляхом застосування підходу усереднення. За умови ергодичності відповідного марковського процесу початкова стохастична система в граничному випадку може бути замінена детермінованою моделлю, параметри якої визначаються стаціонарним розподілом середовища. Така редукція зберігає основні якісні властивості системи та водночас спрощує її аналіз.

Поведінку моделі проілюстровано на основі кількох сценаріїв, що відображають різні технологічні середовища, зокрема інноваційну адаптацію, технологічну поляризацію та нестабільні режими перемикання. Результати моделювання показують, що в більшості випадків траєкторії стохастичної системи залишаються сконцентрованими поблизу відповідної усередненої динаміки, що підтверджує застосовність підходу усереднення. Водночас у асиметричних середовищах із вираженими нелінійними ефектами можуть виникати помітні відхилення. Запропонована модель може бути використана для дослідження довгострокової взаємодії між автоматизацією та зайнятістю, а також для оцінювання структурних змін технологічного ринку праці в умовах невизначеності.

Ключові слова: *стохастичні системи, метод усереднення, марковські перемикання, технологічний ринок праці.*