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A CRITERION FOR THE EQUIVALENCE OF EXPONENT MATRICES

The paper establishes a criterion for the equivalence of reduced exponent matrices in terms of weighted admissible quivers. Exponent matrices arise naturally in the theory of tiled orders over discrete valuation rings and determine a number of structural properties of such orders, including their quivers. Therefore, the problem of recognizing when two reduced exponent matrices are equivalent is important both for the classification of algebraic objects and for the comparison of their associated graph models.

The article proves that two reduced exponent matrices are equivalent if and only if their admissible quivers are isomorphic and the weights of the corresponding simple cycles coincide. This result strengthens the known necessary invariants of equivalence, such as equality of the sums of matrix entries and isomorphism of quivers, which are not sufficient by themselves. The criterion is formulated in the language of weight functions and cycle weights, which makes it convenient for use in combinatorial and graph-theoretic analysis of algebraic data.

From the viewpoint of mathematical modelling, an exponent matrix may be interpreted as an integer-valued directed distance model satisfying triangle-type constraints. In this interpretation, elementary transformations preserve cycle weights and correspond to gauge-type changes of vertex potentials. Thus the obtained criterion can be used to identify equivalent weighted directed graph models, to reduce redundant representations in algebraic modelling, and to compare discrete

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structures that arise in network models, optimization problems, representation-theoretic constructions, and other discrete models whose meaningful invariants are encoded by directed cycles. The result also gives a practical test for comparing representations without reconstructing the corresponding tiled orders.

Key words: *exponent matrix, reduced exponent matrix, admissible quiver, weighted quiver, equivalent exponent matrices, simple cycle, mathematical modelling.*

Introduction. One of the important directions in ring theory is the study of rings by means of graph-theoretic methods. In particular, tiled orders over discrete valuation rings are described by their exponent matrices, and many structural properties of such orders, including their quivers, are determined by these matrices [1-3].

During the last decades exponent matrices have also become an independent object of study. Their associated admissible quivers, unit quivers and special Gorenstein cases have been investigated in a series of works [4-10]. A more recent perspective relates integral quasi-semimetrics, polyhedral cones and max-plus algebra to exponent matrices of tiled orders, which shows that these objects naturally occur in combinatorial and algebraic modelling [11].

In mathematical modelling, an exponent matrix can be interpreted as an integer-valued directed distance table: its entries assign weights to ordered pairs of vertices and satisfy inequalities analogous to the triangle inequality. In this sense, exponent matrices provide compact models of weighted directed networks with algebraic constraints. The equivalence relation considered in this paper identifies matrices that represent the same underlying weighted-quiver structure up to elementary transformations and a simultaneous relabelling of vertices.

From the viewpoint of exponent-matrix theory, equivalent matrices may be regarded as the same object. It is therefore important to have an effective criterion for equivalence. It is known that equivalent exponent matrices have equal sums of entries and isomorphic quivers; however, these conditions are not sufficient. The present paper gives a criterion in the language of weight functions on admissible quivers.

The main result states that two reduced exponent matrices are equivalent precisely when their quivers are isomorphic and the weights of corresponding simple cycles are equal. This formulation makes the equivalence problem transparent in graph-theoretic terms and connects the algebraic transformations of exponent matrices with invariants of weighted directed cycles.

Definition 1. A matrix $\mathscr{E} = (\alpha_{ij}) \in M_n(\mathbb{Z})$ (where $M_n(\mathbb{Z})$ is the ring of n by n matrices with integer entries) satisfying the following conditions:

1) $\alpha_{ij} + \alpha_{jk} \geq \alpha_{ik}$ for all $i, j, k = 1, ..., n$,
2) $\alpha_{ii} = 0$ for all $i = 1, ..., n$,

is called an *exponent matrix*. An exponent matrix satisfying the condition

3) $\alpha_{ij} + \alpha_{ji} \geq 1$ for all $i, j \in \{1, ..., n\}$ $(i \neq j)$

is called a *reduced exponent matrix*.

Let $\mathscr{E} = (\alpha_{ij})$ be a reduced exponent matrix. Introduce the matrix $\mathscr{E}^{(1)} = (\beta_{ij}) = \mathscr{E} + E_n \in M_n(\mathbb{Z})$, where E_n is the identity matrix. Introduce the matrix $\mathscr{E}^{(2)} = (\gamma_{ij}) \in M_n(\mathbb{Z})$: $\gamma_{ij} = \min\limits_{1\leq k\leq n}\left(\beta_{ik} + \beta_{kj}\right)$.

Definition 2. The *quiver* of a reduced exponent matrix $Q = Q(\mathscr{E})$ is the quiver whose adjacency matrix is defined by the formula $[Q] = \mathscr{E}^{(2)} - \mathscr{E}^{(1)}$.

Theorem 1. *If $\mathscr{E}$ is a reduced exponent matrix and $Q = Q(\mathscr{E})$ is the quiver of the exponent matrix, then* $[Q]$ *is a* $(0, 1)$*-adjacency matrix of a strongly connected quiver.*

Definition 3. Reduced exponent matrices $\mathscr{E}_1$ and $\mathscr{E}_2$ are called *equivalent* if one can be obtained from the other by elementary transformations of the following two types:

1) subtract an integer t from all entries of the *i*-th row and add the same integer to all entries of the *i*-th column;
2) interchange two rows and, simultaneously, interchange the two columns with the same numbers.

Definition 4. A quiver Q is called *admissible* if there exists a reduced exponent matrix $\mathscr{E}$ such that $Q(\mathscr{E}) = Q$.

Definition 5. A quiver $Q = \left(V_Q, A_Q\right)$ is called *weighted* if a function $\omega: A_Q \to R$ is defined. The function ω is called a weight function, and its value on an arrow is called the weight of that arrow. If $Q = Q\left(E\right) = \left(q_{ij}\right)$, then $\omega\left(q_{ij}\right) = \alpha_{ij}$.

The sum of the weights of all arrows in a path is called the weight of the path.

Theorem 2. *A strongly connected quiver $Q = \left(V_Q, A_Q\right)$ is admissible if and only if there exists a weight function $\omega: A_Q \to N \cup \{0\}$ satisfying the following conditions:*

1) *the weight of an arrow from vertex i to vertex j is less than the weight of any path from i to j of length* $l \geq 2$;
2) *the weight of a loop at vertex i is less than the weight of any cycle passing through vertex i and having length* $l \geq 2$;
3) *the weight of every cycle is greater than or equal to* 1;
4) *the weight of a loop is equal to* 1;
5) *through every vertex without a loop there passes a cycle of length* $l \geq 2$ *whose weight is equal to* 1 [4].

Corollary. By conditions (4) and (5), through every vertex of an admissible quiver there passes a cycle of weight 1.

Definition 6. A simple cycle in a quiver $Q=\left(V_Q,A_Q\right)$ whose weight is equal to 1 will be called a *unit cycle*.

Proposition 2. In an admissible quiver $Q=\left(V_Q,A_Q\right)$, there are no arrows between the vertices of a unit cycle except the arrows of this cycle.

Lemma 1. *Let* $\mathscr{E}=(\alpha_{ij})\in M_n(\mathbb{Z})$ *be equivalent to* $A=(a_{ij})$ *by transformations of the first type only. Then*

$$\alpha_{i1i_2}+\alpha_{i2i_3}+\ldots+\alpha_{ik-1i_k}+\alpha_{iki_1}=a_{i1i_2}+a_{i2i_3}+\ldots+a_{ik-1i_k}+a_{iki_1}.$$

Proof. An arbitrary transformation of the first type has the form

$$a_{ij}=\alpha_{ij}+t_i-t_j$$

for some set of integers t_1, t_2, ..., t_n. Then

$$a_{i1i_2}+a_{i2i_3}+\ldots+a_{ik-1i_k}+a_{iki_1}=\left(\alpha_{i_1i_2}+t_{i_1}-t_{i_2}\right)+a_{i2i_3}+\ldots+a_{ik-1i_k}+a_{iki_1}.$$

Without loss of generality, assume that an elementary transformation of the first type changes the element α_{i1i_2} (the argument for the other elements is analogous). If t is subtracted from the i_1-th row of the matrix $\mathscr{E}$ and added to the i_1-th column, then the element α_{i1i_2} decreases by *t*, whereas the element α_{iki_1} increases by *t*; hence the sum $\alpha_{i1i_2}+\alpha_{i2i_3}+\ldots+\alpha_{ik-1i_k}+\alpha_{iki_1}$ does not change. Similarly, if t is subtracted from the i_2-th row of the matrix $\mathscr{E}$ and added to the i_2-th column, then the element α_{i2i_3} decreases by *t*, whereas the element α_{i1i_2} increases by *t*; hence the sum $\alpha_{i1i_2}+\alpha_{i2i_3}+\ldots+\alpha_{ik-1i_k}+\alpha_{iki_1}$ does not change. **The lemma is proved.**

Corollary. An elementary transformation does not change the sum of the entries along a cycle.

Theorem 3. *If the matrix* $\mathscr{E}_2=(\alpha_{ij})\in M_n(\mathbb{Z})$ *can be obtained from the matrix* $\mathscr{E}_1=(r_{ij})\in M_n(\mathbb{Z})$ *by elementary transformations of the first type and the first rows of* $\mathscr{E}_1$ *and* $\mathscr{E}_2$ *coincide, then* $\mathscr{E}_1=\mathscr{E}_2$.

Proof. By Lemma 1, elementary transformations of the first type do not change the sum of the entries of a cycle: $\alpha_{12}+\alpha_{21}=r_{12}+r_{21}$. In addition, $\alpha_{12}=r_{12}$ (the first rows of the matrices coincide), and therefore $\alpha_{21}=r_{21}$. Similarly, $\alpha_{j1}=r_{j1}$, for $j=3,\ldots,n$. Hence, the first columns of the matrices $\mathscr{E}_1$, $\mathscr{E}_2$ coincide. Analogously, we have

$$\alpha_{1j}+\alpha_{jk}+\alpha_{k1}=r_{1j}+r_{jk}+r_{k1},\alpha_{1j}=r_{1j},\alpha_{k1}=r_{k1},$$

whence $\alpha_{jk}=r_{jk}$ for $j,k=2,\ldots,n$. Thus $\mathscr{E}_1=\mathscr{E}_2$. **The theorem is proved.**

Definition 7. For a set M of entries of an exponent matrix $E=(\alpha_{ij})\in M_n(Z)$, one can construct an undirected graph

$$G(M):V_G=\{1,2,\ldots,n\},A_G=\{\sigma_{ij}\mid 1\le i<j\le n,\alpha_{ij}\in M \text{or} \alpha_{ji}\in M\}.$$

For two exponent matrices $\mathscr{E}_1=(r_{ij})$ and $\mathscr{E}_2=(\alpha_{ij})$ from $M_n(\mathbb{Z})$, consider a set M that contains only those entries of the matrices for which $r_{ij}=\alpha_{ij}$ (but not necessarily all such entries).

Theorem 4. *Let the matrix* $\mathscr{E}_2=(\alpha_{ij})\in M_n(\mathbb{Z})$ *be obtainable from the matrix* $\mathscr{E}_1=(r_{ij})\in M_n(\mathbb{Z})$ *by elementary transformations of the first type. If* $G(M)$ *is connected, then* $\mathscr{E}_1=\mathscr{E}_2$; *if* $G(M)$ *is not connected, then it is possible that* $\mathscr{E}_1\neq\mathscr{E}_2$.

Proof. Let $G(M)$ be connected and

$$\alpha_{ij}=r_{ij}\in M\ . \tag{1}$$

By Lemma 1, $\alpha_{ij}+\alpha_{ji}=r_{ij}+r_{ji},$; therefore

$$\alpha_{ji}=r_{ji}\,. \tag{2}$$

We prove that $\alpha_{xy}=r_{xy}$ for arbitrary $x, y\in\{1, ..., n\}$. Fix x and y. Since $G(M)$ is connected, there is a simple path beginning at vertex y and ending at vertex x. Let this path be ($y=i_1, i_2, ..., i_k=x$). By Lemma 1,

$$\alpha_{i1i_2}+\alpha_{i2i_3}+\ldots+\alpha_{ik-1i_k}+\alpha_{iki_1}=r_{i1i_2}+r_{i2i_3}+\ldots+r_{ik-1i_k}+r_{iki_1}\,. \tag{3}$$

where $\alpha_{i1i_2}\in M$ or $\alpha_{i2i_1}\in M$. Hence, from (1) or (2), it follows that $\alpha_{i1i_2}=r_{i1i_2}$.

Similarly,

$$\alpha_{i2i_3}=r_{i2i_3},\ldots,\alpha_{ik-1i_k}=r_{ik-1i_k}\,. \tag{4}$$

From (3) and (4) we obtain $\alpha_{iki_1}=r_{iki_1}$, that is, $\alpha_{xy}=r_{xy}$. Thus $\mathscr{E}_1=\mathscr{E}_2$.

Let G be disconnected. Then G decomposes into connected components. Let the first connected component consist of the vertices $i_1, i_2, ..., i_k$. If, in the matrix $\mathscr{E}_1$, elementary transformations of the first type are performed for rows and columns with numbers $i_1, i_2, ..., i_k$ by subtracting a number t from the i_p-th row and adding it to the i_p-th column for all $p\in\{1, ..., k\}$, then the entries lying at the intersections of these rows and columns do not change, since $\alpha_{ij}=r_{ij}+t_i-t_j$ and $t_i=t_j=t$. Since G is disconnected, the entries of M that do not belong to the first connected component also do not change under these elementary transformations. If some entry not belonging to the first component changed, then G would contain an edge between the first component and a vertex of another component, which is impossible. Thus the entries of M remain unchanged, while the exponent matrix before the transformation differs from the exponent matrix after the transformation. **The theorem is proved.**

The next lemma is a generalization of Lemma 1.

Lemma 2. *Let* $i_1, i_2, \ldots, i_k$ *be an ordered set of indices, and let a set M of k elements satisfy the following condition: for each t, either* $\alpha_{i_t i_{t+1}} \in M$ *or* $\alpha_{i_{t+1} i_t} \in M$ *(we assume that* $i_{k+1} = i_1$*).*

Let $a_t = \alpha_{i_t i_{t+1}}$ *if* $\alpha_{i_t i_{t+1}} \in M$, and $a_t = \alpha_{i_{t+1} i_t}$ if $\alpha_{i_{t+1} i_t} \in M$; $s_t = 1$ if $\alpha_{i_t i_{t+1}} \in M$, and $s_t = -1$ if $\alpha_{i_{t+1} i_t} \in M$.

Then transformations of the first type do not change the sum

$$s_1 a_1 + s_2 a_2 + \ldots + s_k a_k. \tag{5}$$

Proof. Denote $S_{ij} = \alpha_{ij} + \alpha_{ji}$. Transform the sum $s_1 a_1 + s_2 a_2 + \ldots + s_k a_k$ as follows: the terms for which $s_i = 1$ are left unchanged, $\alpha_{i_t i_{t+1}}$, whereas the terms for which $s_i = -1$ are transformed:

$$(-1)a_t = (-1)\alpha_{i_{t+1} i_t} = (-1)\left(S_{i_t i_{t+1}} - \alpha_{i_t i_{t+1}}\right) = \alpha_{i_t i_{t+1}} - S_{i_t i_{t+1}}.$$

After the transformation, the sum (5) has the form

$$\sum_{t: s_t = 1} \alpha_{i_t i_{t+1}} - \sum_{t: s_t = -1} S_{i_t i_{t+1}}$$

By Lemma 1, transformations of the first type do not change the sum

$$\sum_{t=1}^{k} \alpha_{i_t i_{t+1}}$$

and the sums $S_{i_t i_{t+1}}$. The lemma is proved.

Example 1. Let $\mathscr{E} = \begin{pmatrix} 0 & 2 & 2 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 2 & 2 & 0 \end{pmatrix}$, $M = \{\alpha_{12}, \alpha_{13}, \alpha_{42}, \alpha_{43}\}$, and let the ordered set of indices be {1, 2, 4, 3}. Then $a_1 = \alpha_{12}$, $s_1 = 1$, $a_2 = \alpha_{42}$, $s_2 = -1$, $a_3 = \alpha_{43}$, $s_3 = 1$, $a_4 = \alpha_{13}$, $s_4 = -1$. By Lemma 2, transformations of the first type do not change the expression $\alpha_{12} - \alpha_{42} + \alpha_{43} - \alpha_{13}$.

Example 2. Let $\mathscr{E} = \begin{pmatrix} 0 & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & 0 & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & 0 \end{pmatrix}$, $M = \{\alpha_{12}, \alpha_{13}, \alpha_{23}\}$ and let the ordered set of indices be {1, 2, 3}. Then $a_1 = \alpha_{12}$, $s_1 = 1$, $a_2 = \alpha_{23}$, $s_2 = 1$, $a_3 = \alpha_{13}$, $s_3 = -1$, and by Lemma 2 transformations of the first type do not change the expression $\alpha_{12} + \alpha_{23} - \alpha_{13}$.

Theorem 5. *The entries of a set M of an exponent matrix* $\mathscr{E} = (\alpha_{ij}) \in M_n(\mathbb{Z})$*, which do not lie on the main diagonal, can be transformed into arbitrary integers by transformations of the first type if and only if the graph* $G(M)$ *is acyclic.*

Proof. If the graph $G(M)$ contains a cycle, then by Lemma 2 transformations of the first type do not change the sum $s_1a_1 + s_2a_2 + \ldots + s_ka_k$.

Therefore, in the general case, not all entries of the matrix corresponding to a cycle in G can be transformed into arbitrary integers.

Conversely, assume that G is acyclic. Then G is a forest, i.e. it consists of trees. Consider the tree containing the first vertex and take vertex 1 as its root. Let N_1 be the set of vertices at the first level, i.e. vertices connected with the root. Let N_2 be the set of vertices whose shortest path to the root has length 2. Similarly define N_3, ..., N_k. Notice that every vertex of the tree except the root is connected with vertices of the preceding level by exactly one edge; otherwise a cycle would exist. For each vertex $x \in N_1$ perform a transformation of the first type: subtract α_{1x} from the entries of the x-th column and add this number to the entries of the x-th row. For each vertex $y \in N_2$ perform a transformation of the first type: subtract α_{xy} from the entries of the y-th column (where $x \in N_1$ is the vertex of the first level connected with y) and add this number to the entries of the y-th row. Continuing this process through all levels and then through all trees, the entries of M become equal to zero. By the same argument, the entries of M can be transformed into arbitrary integers. **The theorem is proved.**

Theorem 6. *An exponent matrix $\mathscr{E}_2$ can be obtained from an exponent matrix $\mathscr{E}_1$ by elementary transformations if and only if the quiver $Q(\mathscr{E}_1)$ is isomorphic to the quiver $Q(\mathscr{E}_2)$ and the weights of the corresponding simple cycles of $Q(\mathscr{E}_1)$ and $Q(\mathscr{E}_2)$ are equal.*

Proof. A transformation of the second type of a reduced exponent matrix corresponds to a renumbering of the vertices of the quiver Q. Transformations of the first type do not change the quiver. Hence, by transformations of the second type, the matrix $\mathscr{E}_1$ can be transformed into a matrix $\mathscr{E}_{11}$ such that $Q(\mathscr{E}_{11}) = Q(\mathscr{E}_2)$.

Let Q_1 be the weighted quiver determined by the exponent matrix $\mathscr{E}_{11}$, and let Q_2 be the weighted quiver determined by the exponent matrix $\mathscr{E}_2$.

Consider an arbitrary cycle of Q_1: $v_1 \to v_2 \to \ldots \to v_k \to v_1$. If an integer t is subtracted from the entries of the first row and added to the entries of the first column, then the weight of the arrow (v_1, v_2) decreases by t, while the weight of the arrow (v_k, v_1) increases by t; the weights of the other arrows do not change. Therefore, an elementary transformation of the first type does not change the weight of a cycle.

We prove the converse. Suppose that the weights of corresponding simple cycles of Q_1 and Q_2 coincide. By the corollary to Theorem 2, a vertex of an admissible quiver has no loop if and only if a unit cycle passes through it. By Proposition 2, a unit cycle is simple. Hence, a vertex of a unit quiver has no loop if and only if a simple unit cycle passes through it. Since corresponding simple cycles have equal weights, corresponding vertices of Q_1 and Q_2 either both have a loop or both do not.

Transform $\mathscr{E}_{11}$ into $\mathscr{E}_2$ by transformations of the first type. By Theorem 5, transformations of the first type can make the first row of the matrix $\mathscr{E}_{11}$ zero. Let $\mathscr{E}_{11} = (\alpha_{ij})$ and $\mathscr{E}_2 = (r_{ij})$. By the definition of the weight function, α_{xy} is the minimum weight of a path from x to y, and α_{yx} is the minimum weight of a path from y to x. Therefore, $\alpha_{xy} + \alpha_{yx}$ is the minimum weight of a cycle of the weighted quiver Q_1 passing through x and y. Similarly, $r_{xy} + r_{yx}$ is the weight of the lightest cycle of the weighted quiver Q_2 passing through x and y. Hence

$$\alpha_{xy} + \alpha_{yx} = r_{xy} + r_{yx}. \tag{6}$$

Similarly, $\alpha_{xy} + \alpha_{yz} + \alpha_{zx}$ is the minimum weight of a cycle of the weighted quiver Q_1 passing through the vertices x, y and z. Likewise, $r_{xy} + r_{yz} + r_{zx}$ is the minimum weight of a cycle of the weighted quiver Q_2 passing through the vertices x, y and z. Therefore

$$\alpha_{xy} + \alpha_{yz} + \alpha_{zx} = r_{xy} + r_{yz} + r_{zx}. \tag{7}$$

Let ω_1 and ω_2 be the weight functions corresponding to the exponent matrices $\mathscr{E}_{11}$ and $\mathscr{E}_2$. First change the weight function ω_1 so that the weights of the corresponding arrows of the quivers that begin or end at the first vertex are equal. That is, $\omega_1(\sigma_{1j}) = \omega_2(\sigma_{1j}), \omega_1(\sigma_{j1}) = \omega_2(\sigma_{j1})$, for all arrows σ_{1j} and σ_{j1}.

Write the indices $2 \le j_1, j_2, ..., j_k \le n$ for which there exists an arrow σ_{1j_k} or σ_{j_k1} (possibly both) in increasing order.

1. If the quiver contains the arrow σ_{1j_1}, then from the entries of the column with number j_1 of the matrix $\mathscr{E}_{11}$ one must subtract the number $t = \omega_1(\sigma_{1j_1}) - \omega_2(\sigma_{1j_1})$ and add the number t to the entries of the row with number j_1.
2. If the quiver contains the arrow σ_{j_11}, then from the entries of the row with number j_1 of the matrix $\mathscr{E}_1$ one must subtract the number $t = \omega_1(\sigma_{j_11}) - \omega_2(\sigma_{j_11})$ and add the number t to the entries of the column with number j_1.
3. If the quiver contains the arrow σ_{1j_1} and the arrow σ_{j_11}, then these arrows form a cycle. By the assumption of the theorem,

$$\omega_1(\sigma_{j_11}) + \omega_1(\sigma_{1j_1}) = \omega_2(\sigma_{j_11}) + \omega_2(\sigma_{1j_1});$$

therefore

$$\omega_1(\sigma_{1j_1}) - \omega_2(\sigma_{1j_1}) = \omega_1(\sigma_{j_11}) - \omega_2(\sigma_{j_11}).$$

Hence, in this case transformation 1) is equivalent to transformation 2).

After each transformation, the obtained quiver and exponent matrix are again denoted by Q_1 and $\mathscr{E}_1$.

After transformations 1) or 2) have been performed for all indices $j_1, j_2, ..., j_k$, the weights of the arrows of Q_1 that begin or end at the first vertex are equal to the weights of the corresponding arrows of Q_2.

In exactly the same way, for arrows incident with the second vertex, we use elementary transformations of the first type to change the weight function ω_1 so that the weights of all corresponding arrows beginning or ending at the second vertex are equal.

Write the indices $2 \le i_1, i_2, ..., i_p \le n$ for which there exists an arrow σ_{2i_p} or $\sigma_{i_p 2}$ (possibly both) in increasing order.

1. If the quiver contains the arrow σ_{2i_1}, then from the entries of the column with number i_1 of the matrix $\mathscr{E}_1$ one must subtract the number $t = \omega_1\left(\sigma_{2i_1}\right) - \omega_2\left(\sigma_{2i_1}\right)$ and add the number t to the entries of the row with number i_1.
2. If the quiver contains the arrow $\sigma_{i_1 2}$, then from the entries of the row with number i_1 of the matrix $\mathscr{E}_1$ one must subtract the number $t = \omega_1\left(\sigma_{i_1 2}\right) - \omega_2\left(\sigma_{i_1 2}\right)$ and add the number t to the entries of the column with number i_1.
3. If the quiver contains the arrow σ_{2i_1} and the arrow $\sigma_{i_1 2}$, then transformation 1) is equivalent to transformation 2).

Continuing such transformations, we obtain $\omega_1 = \omega_2$.

Conclusion. Two reduced exponent matrices are equivalent if and only if they have isomorphic quivers and the weights of the corresponding simple cycles are equal.

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КРИТЕРІЙ ЕКВІВАЛЕНТНОСТІ МАТРИЦЬ ПОКАЗНИКІВ

У статті встановлено критерій еквівалентності зведених матриць показників у термінах зважених допустимих сагайдаків. Матриці показників природно виникають у теорії черепичних порядків над дискретно нормованими кільцями та визначають низку структурних властивостей таких порядків, зокрема їхні сагайдаки. Тому задача розпізнавання еквівалентності двох зведених матриць показників є важливою як для класифікації алгебраїчних об’єктів, так і для порівняння пов’язаних із ними графових моделей.

Доведено, що дві зведені матриці показників є еквівалентними тоді і тільки тоді, коли їхні допустимі сагайдаки ізоморфні, а ваги відповідних простих циклів збігаються. Цей результат посилює відомі необхідні інваріанти еквівалентності, зокрема рівність сум елементів матриць та ізоморфізм сагайдаків, які самі по собі не є достатніми. Запропонований критерій сформульовано мовою вагових функцій і ваг циклів, що робить його зручним для комбінаторного та графотеоретичного аналізу алгебраїчних даних.

З погляду математичного моделювання матрицю показників можна інтерпретувати як цілочислову модель напрямлених відстаней, що задовольняє обмеження трикутникового типу. У такій інтерпретації елементарні перетворення зберігають ваги циклів і відповідають калібрувальним змінам вершинних потенціалів. Отже, отриманий критерій можна використовувати для виявлення еквівалентних зважених орієнтованих графових моделей, усунення надлишкових представлень в алгебраїчному моделюванні та порівняння дискретних структур, суттєві інваріанти яких кодуються напрямленими циклами. Зокрема, результат дає практичний тест для порівняння таких представлень без реконструкції відповідних черепичних порядків.

Ключові слова: *матриця показників, зведена матриця показників, черепичний порядок, допустимий сагайдак, зважений сагайдак, простий цикл, критерій еквівалентності, математичне моделювання.*